Ray Casting

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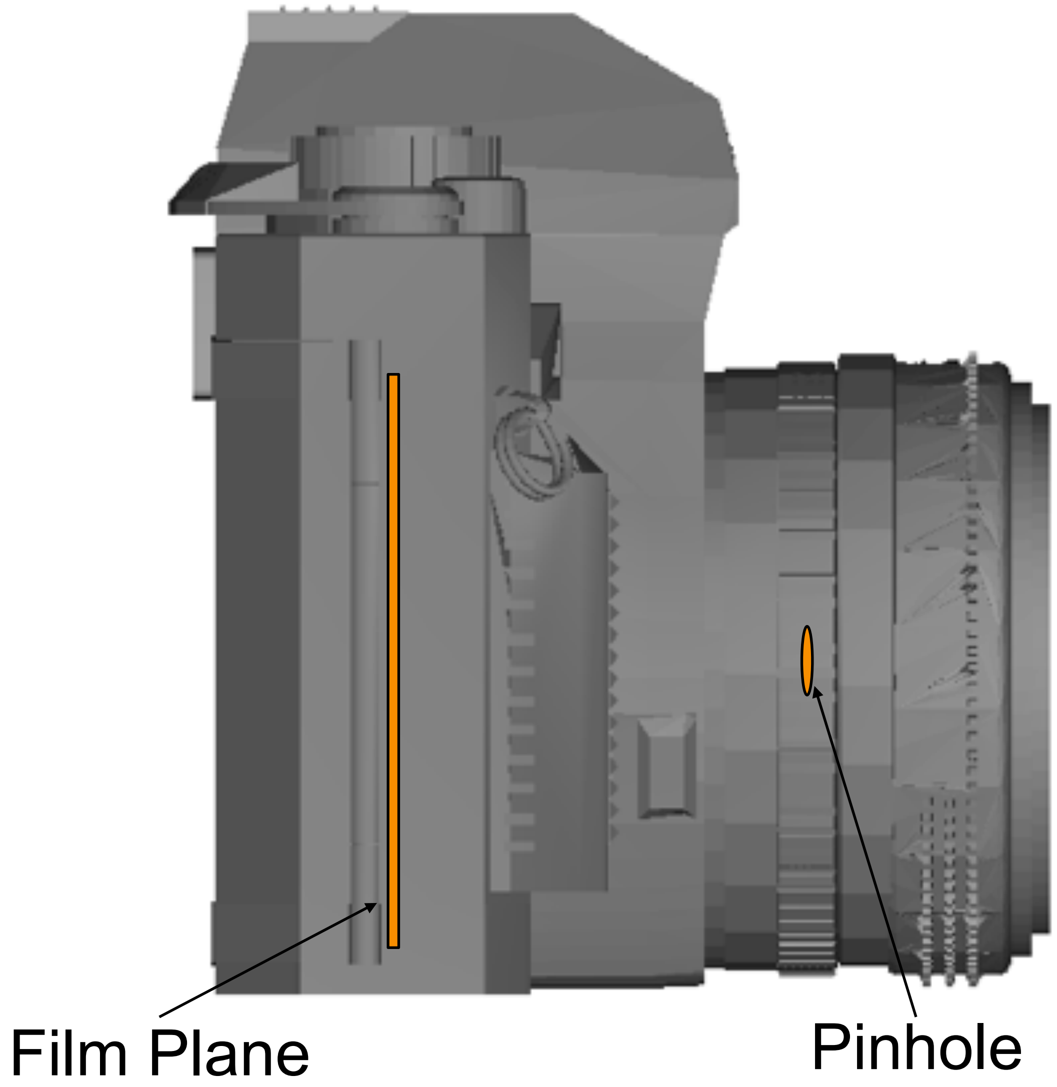
CS 4810: Graphics

Acknowledgment: slides by Jason Lawrence,
Misha Kazhdan, Allison Klein, Tom Funkhouser, Adam Finkelstein and David Dobkin

Traditional Pinhole Camera

- The film sits behind the pinhole of the camera.



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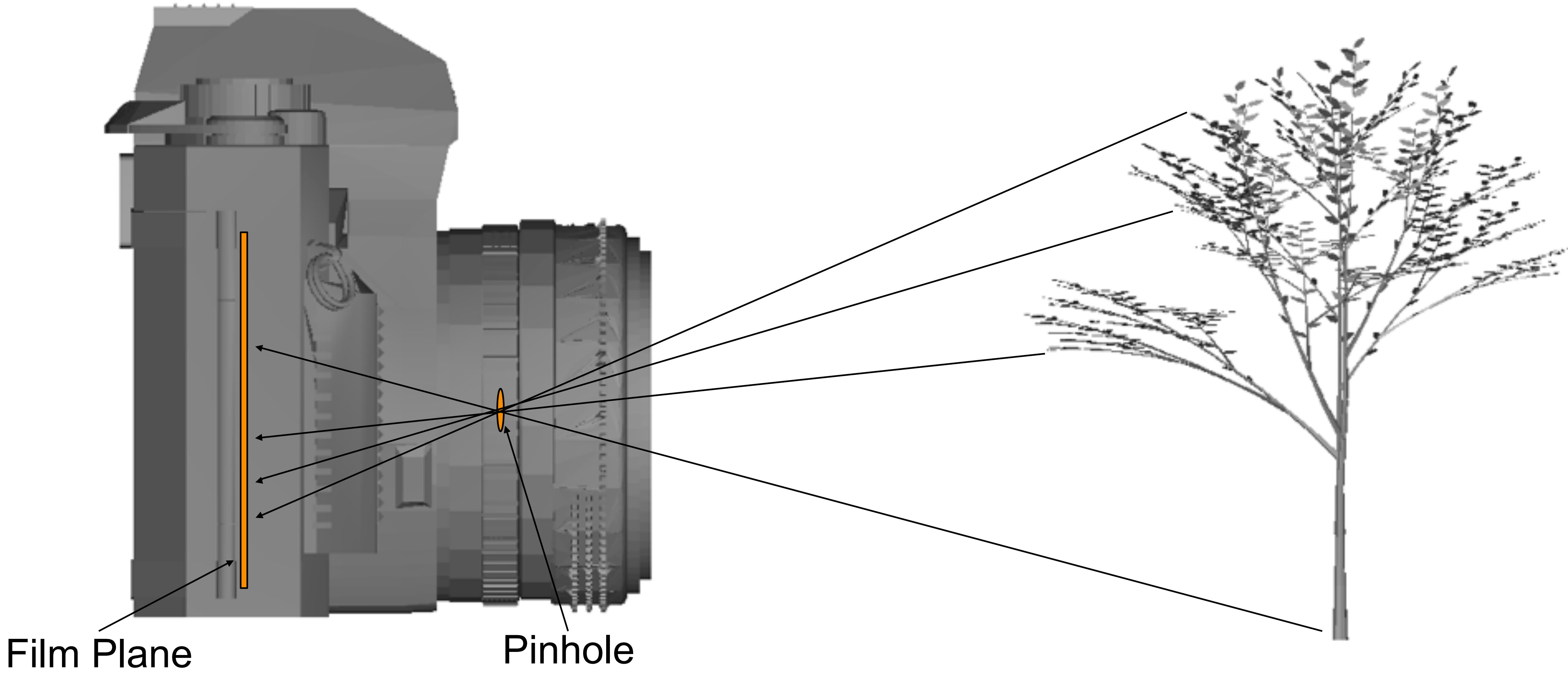
- Rays come in from the outside, pass through the pinhole, and hit the film plane.



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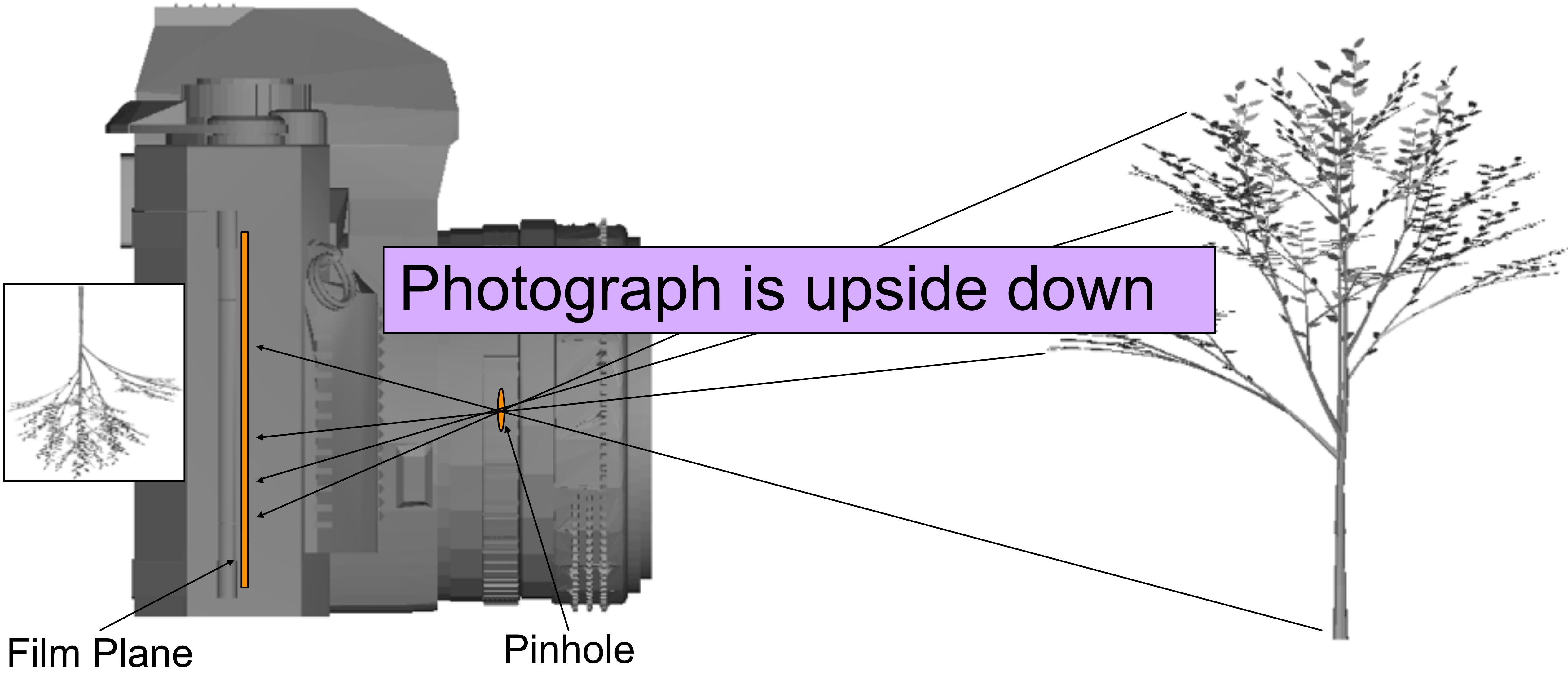
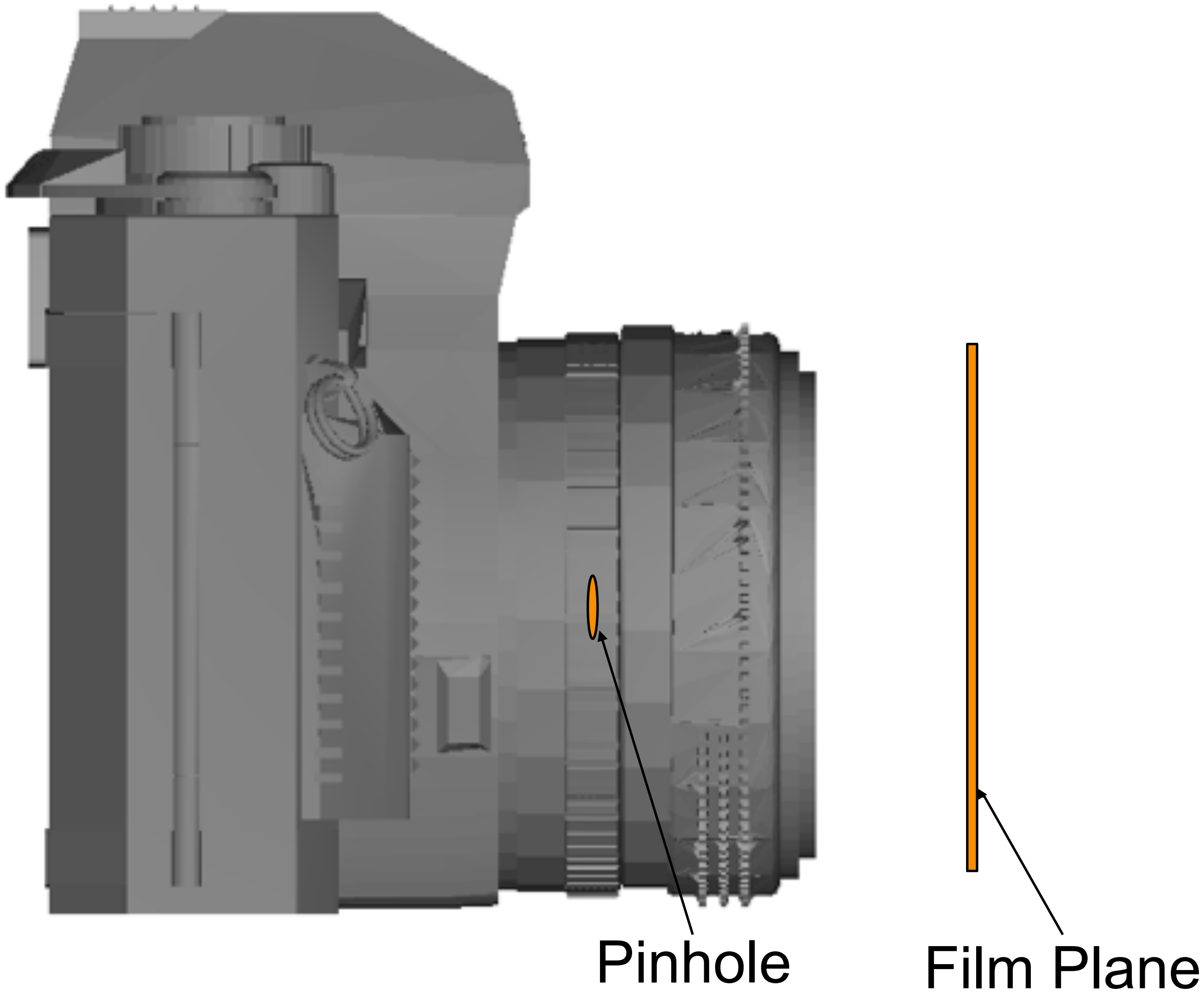


Virtual Camera

- The film sits in front of the pinhole of the camera.



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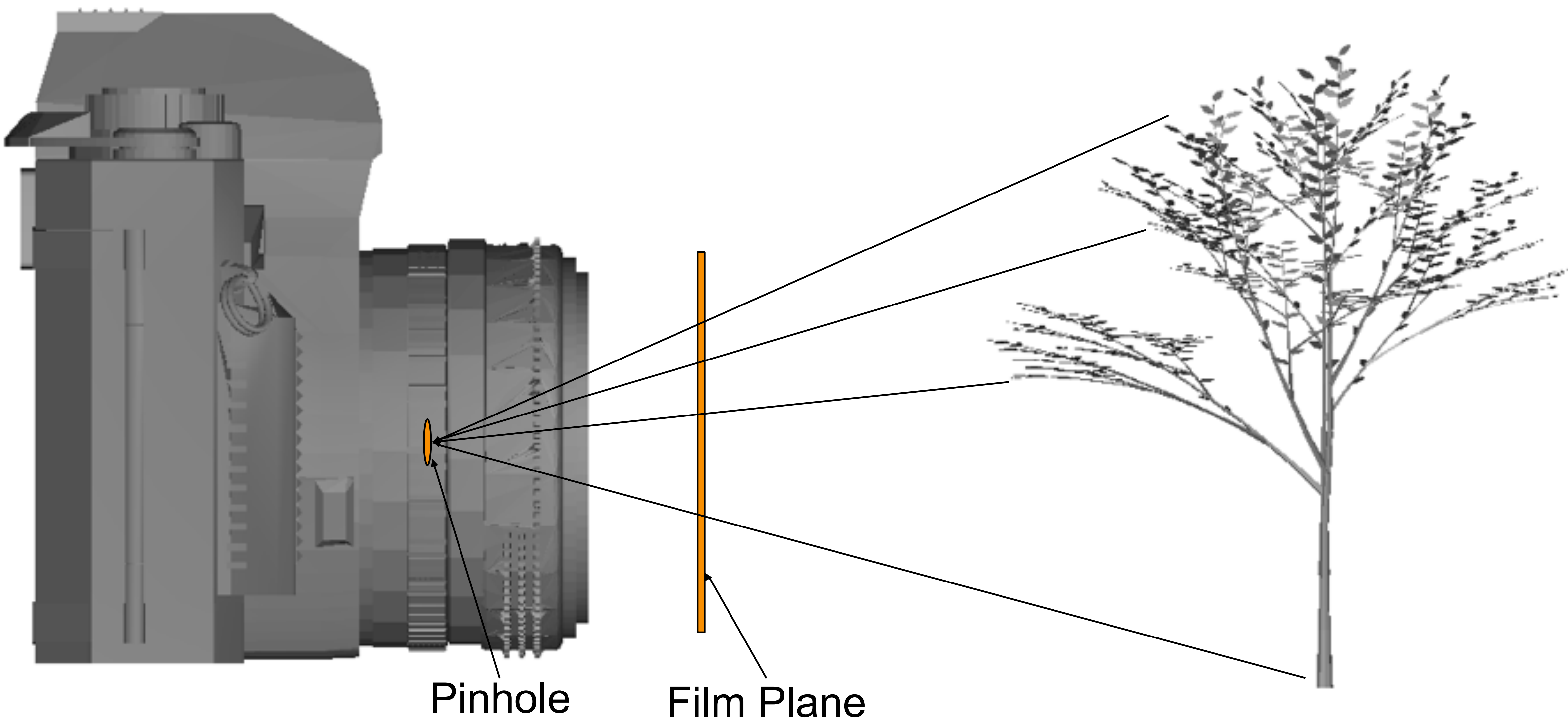

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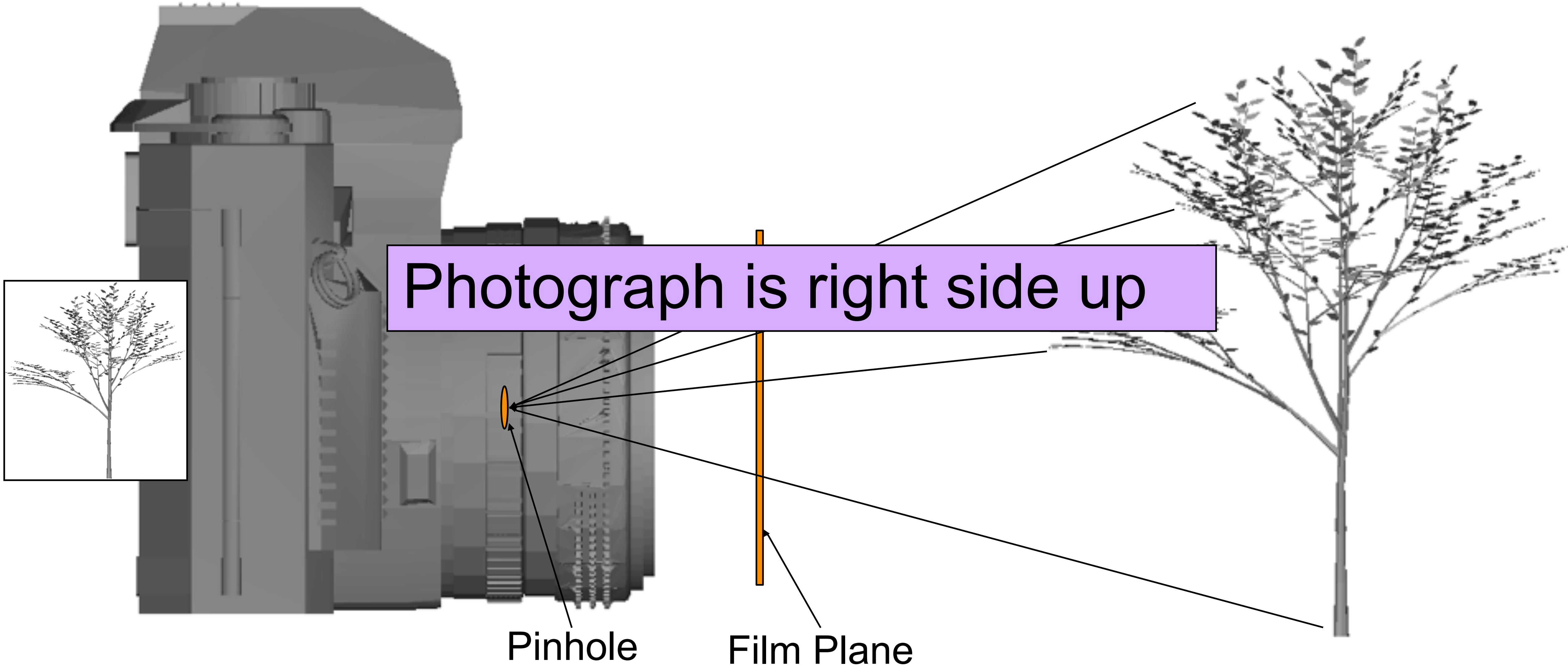
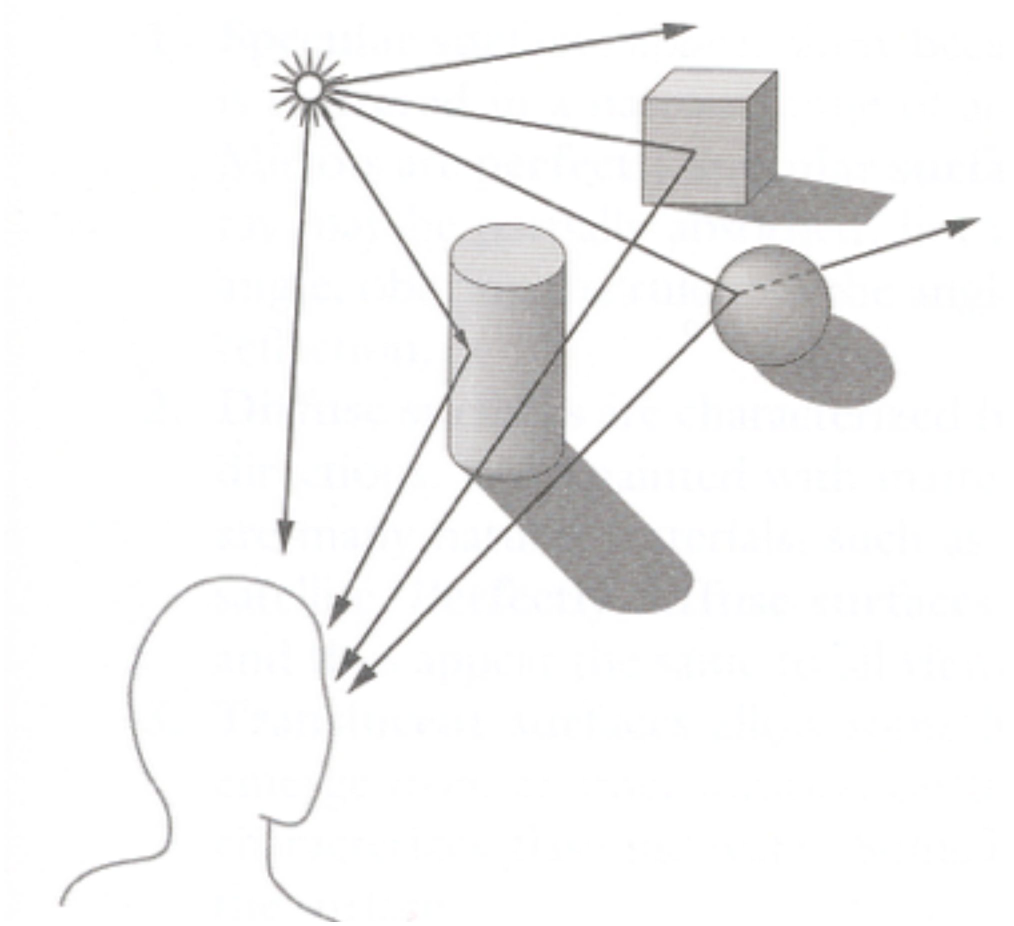


Overview

- Ray Casting
  - o What do we see?
  - o How does it look?

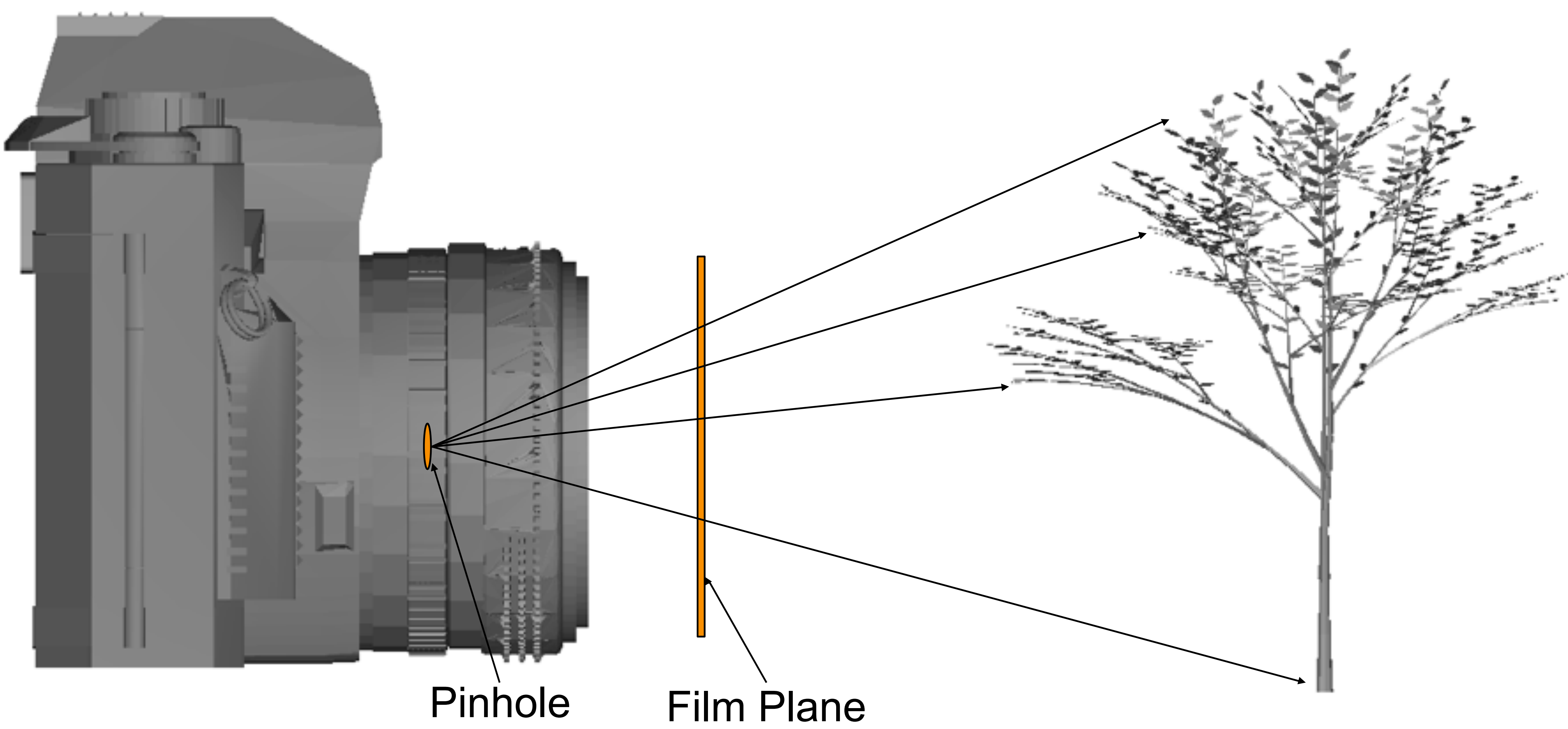


Ray Casting

- Rendering model
- Intersections with geometric primitives
  - Sphere
  - Triangle
- Acceleration techniques
  - Bounding volume hierarchies
  - Spatial partitions
    - Uniform grids
    - Octrees
    - BSP trees

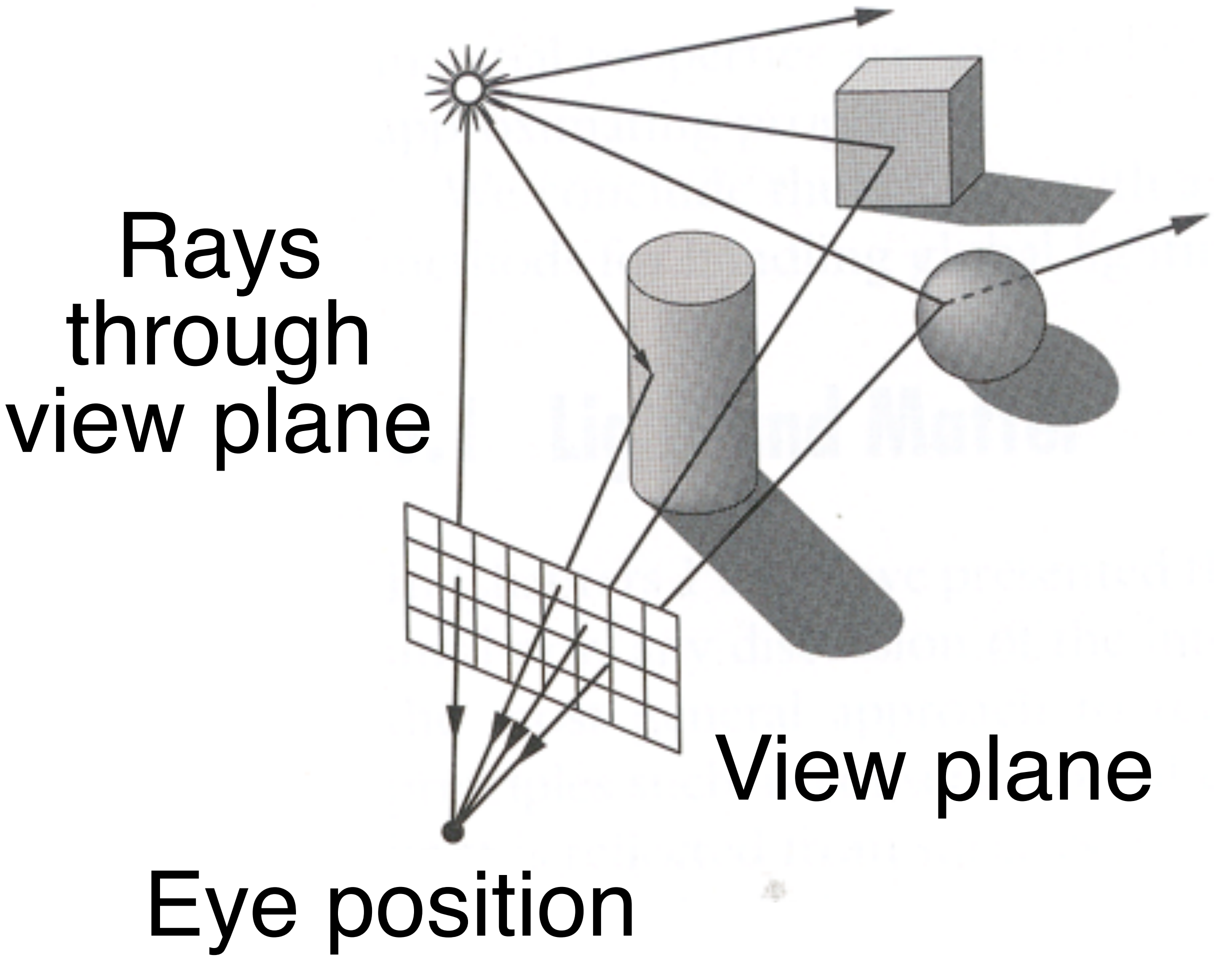
Ray Casting

- We invert the process of image generation by sending rays out from the pinhole, and then we find the first intersection of the ray with the scene.



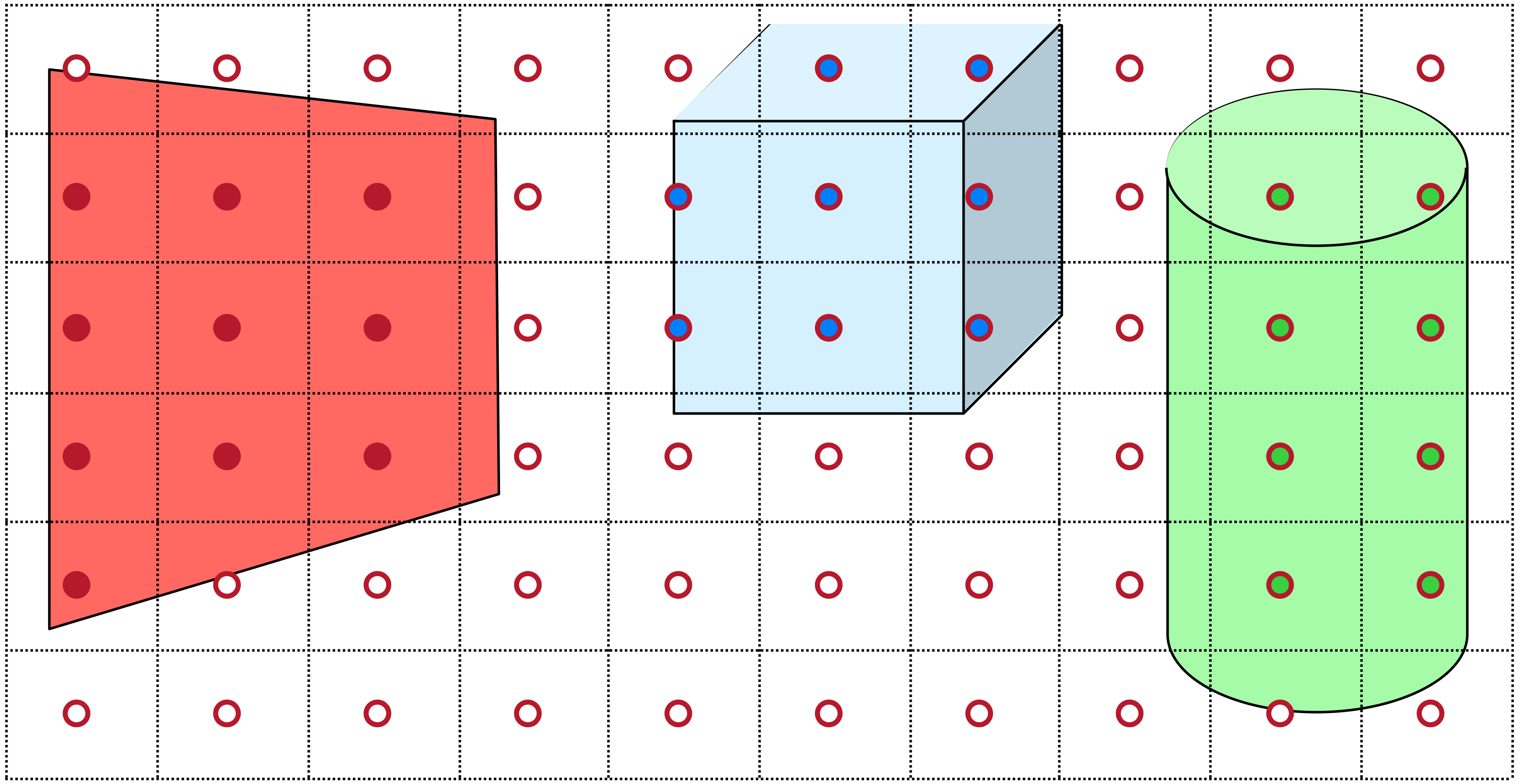
Ray Casting

- The color of each pixel on the view plane depends on the radiance emanating from visible surfaces



Ray Casting

- For each sample …
  - Construct ray from eye position through view plane
  - Find first surface intersected by ray through pixel
  - Compute color sample based on surface radiance



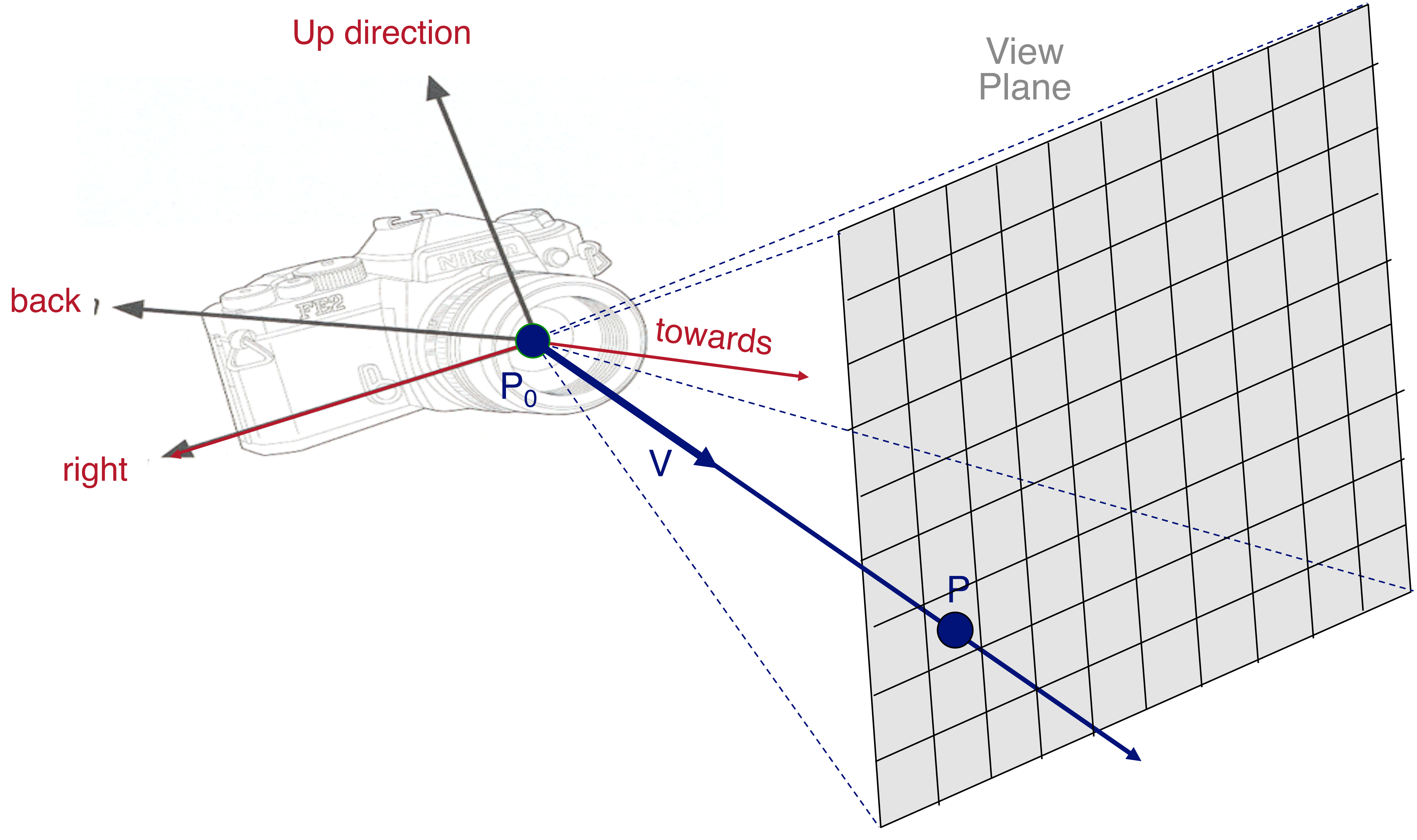
Ray Casting

- Simple implementation:

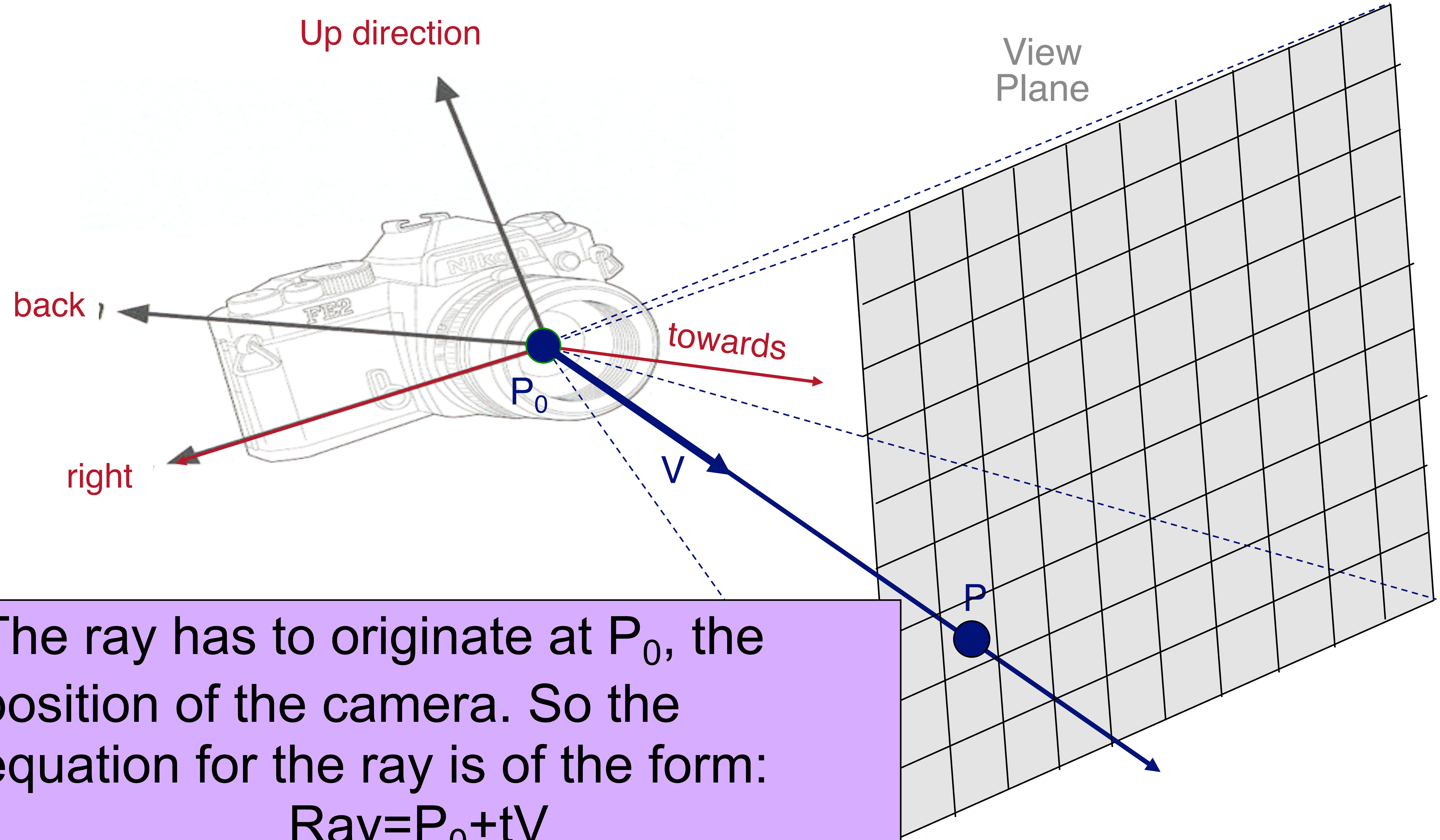
```
Image RayCast(Camera camera, Scene scene, int width, int height)
{
    Image image = new Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            Ray ray = ConstructRayThroughPixel(camera, i, j);
            Intersection hit = FindIntersection(ray, scene);
            image[i][j] = GetColor(hit);
        }
    }
    return image;
}
```

- Where are we looking?
- What are we seeing?
- What does it look like?

Constructing a Ray Through a Pixel



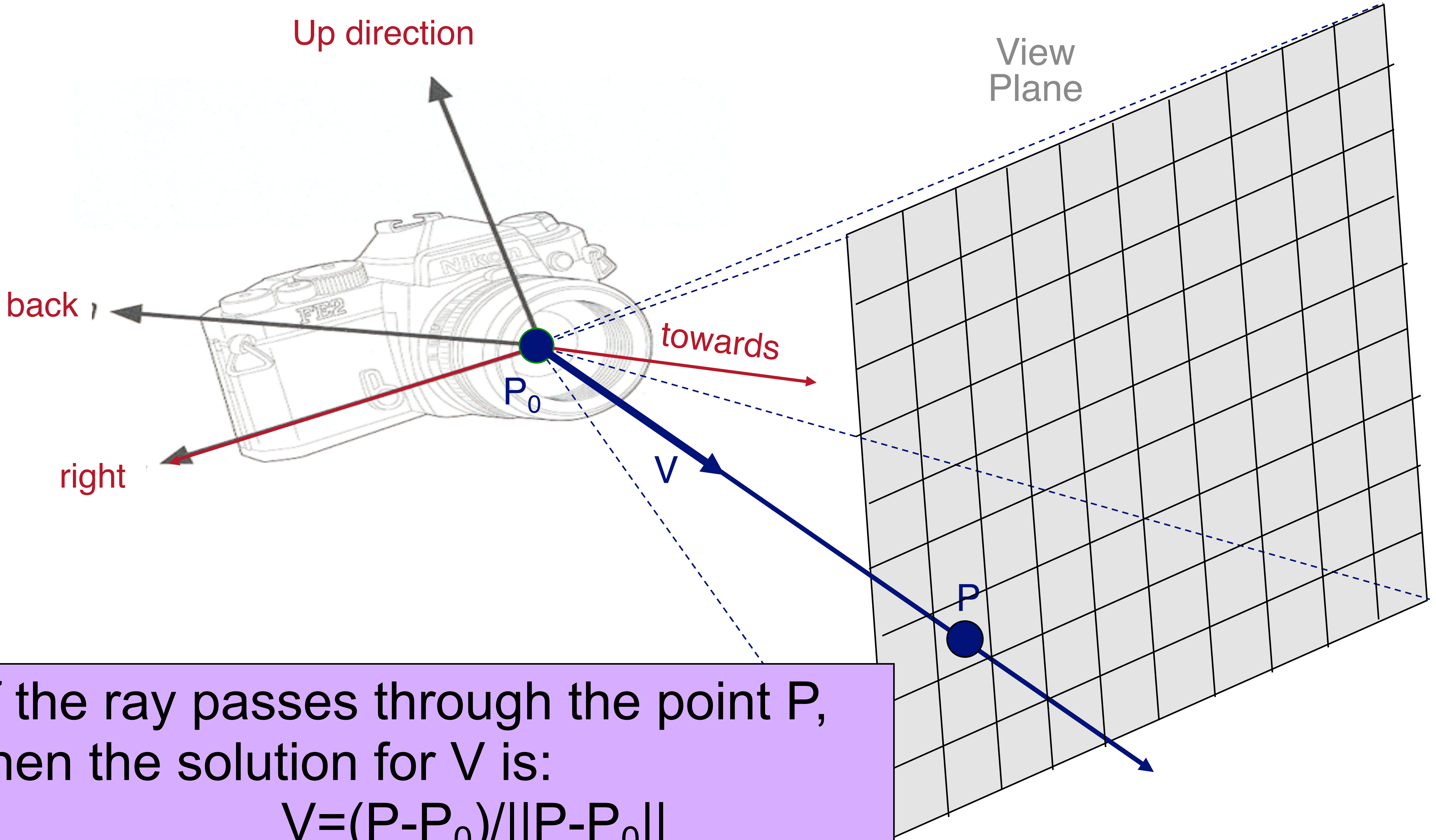
Constructing a Ray Through a Pixel



The ray has to originate at P_0, the position of the camera. So the equation for the ray is of the form:

$$Ray = P_0 + tV$$

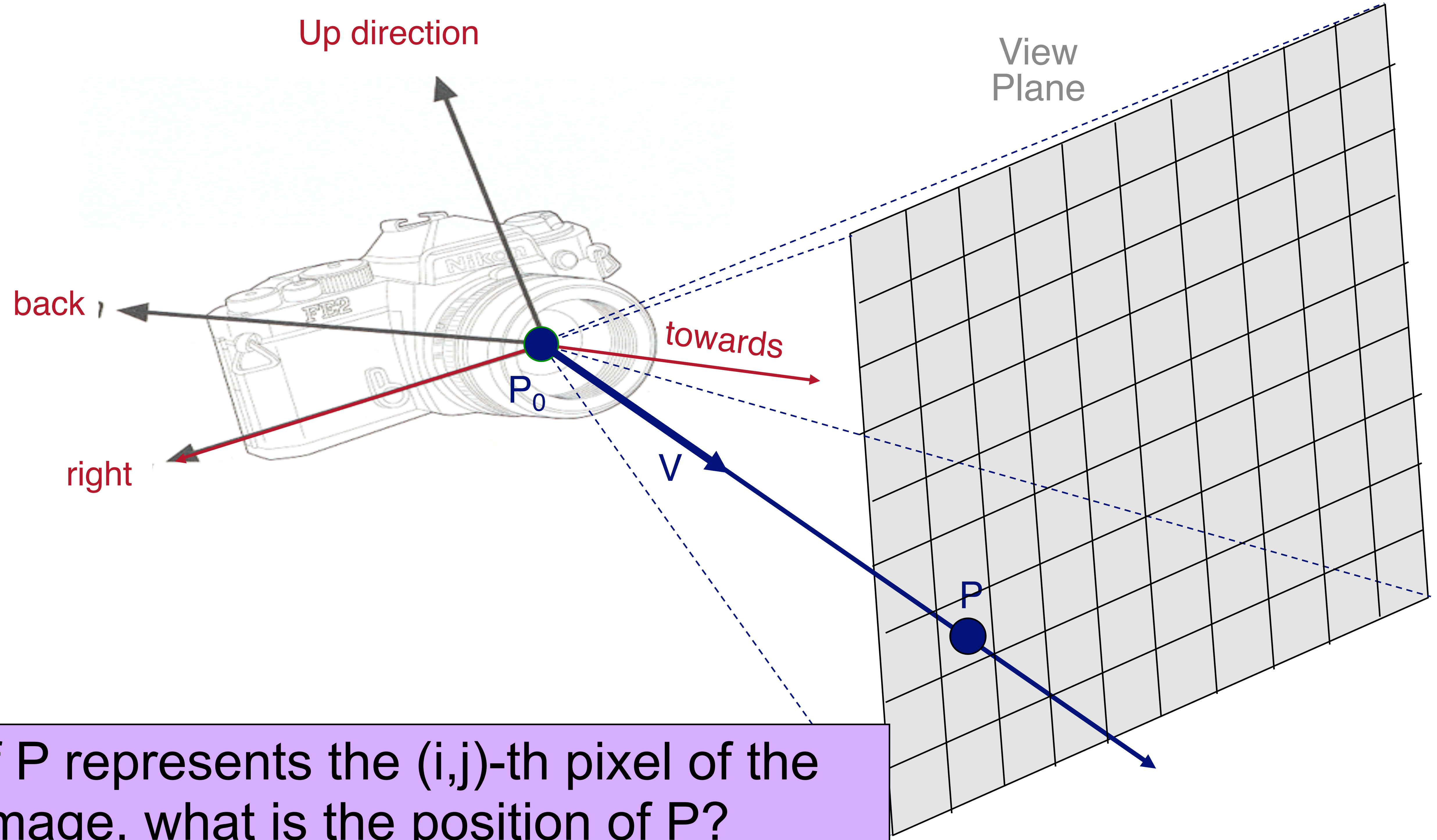
Constructing a Ray Through a Pixel



If the ray passes through the point P, then the solution for V is:

$$V=(P-P_0)/||P-P_0||$$

Constructing a Ray Through a Pixel



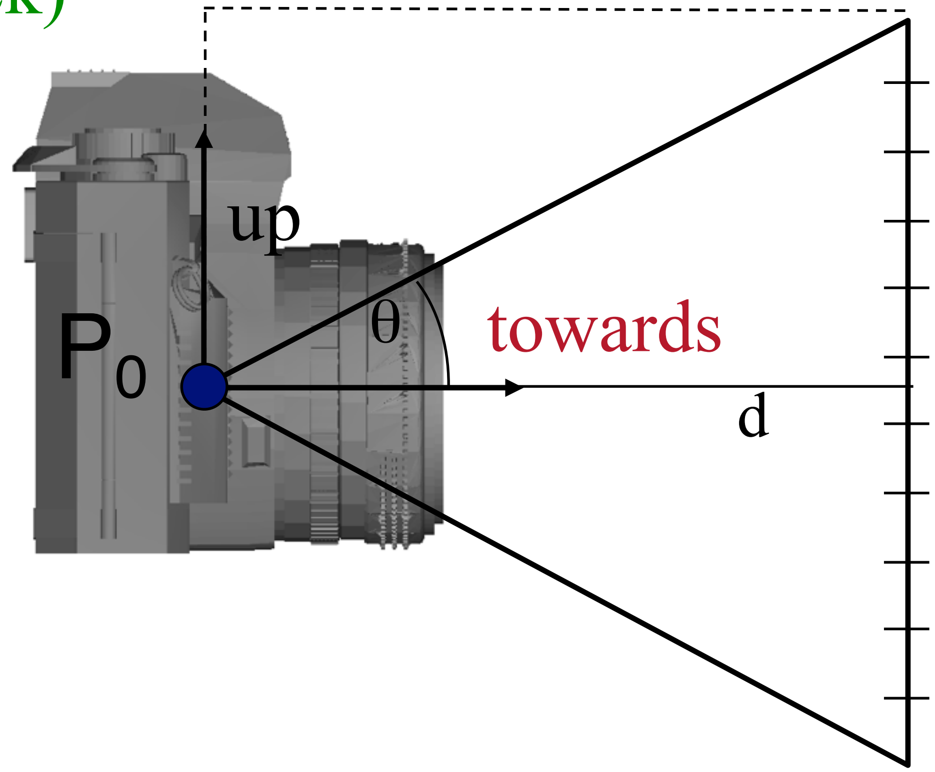
If P represents the (i,j)-th pixel of the image, what is the position of P?

Constructing Ray Through a Pixel

- 2D Example: Side view of camera at P_0
  - What is the position of the *i*-th pixel P[i]?

θ = frustum half-angle (given), or field of view

d = distance to view plane (arbitrary = you pick)



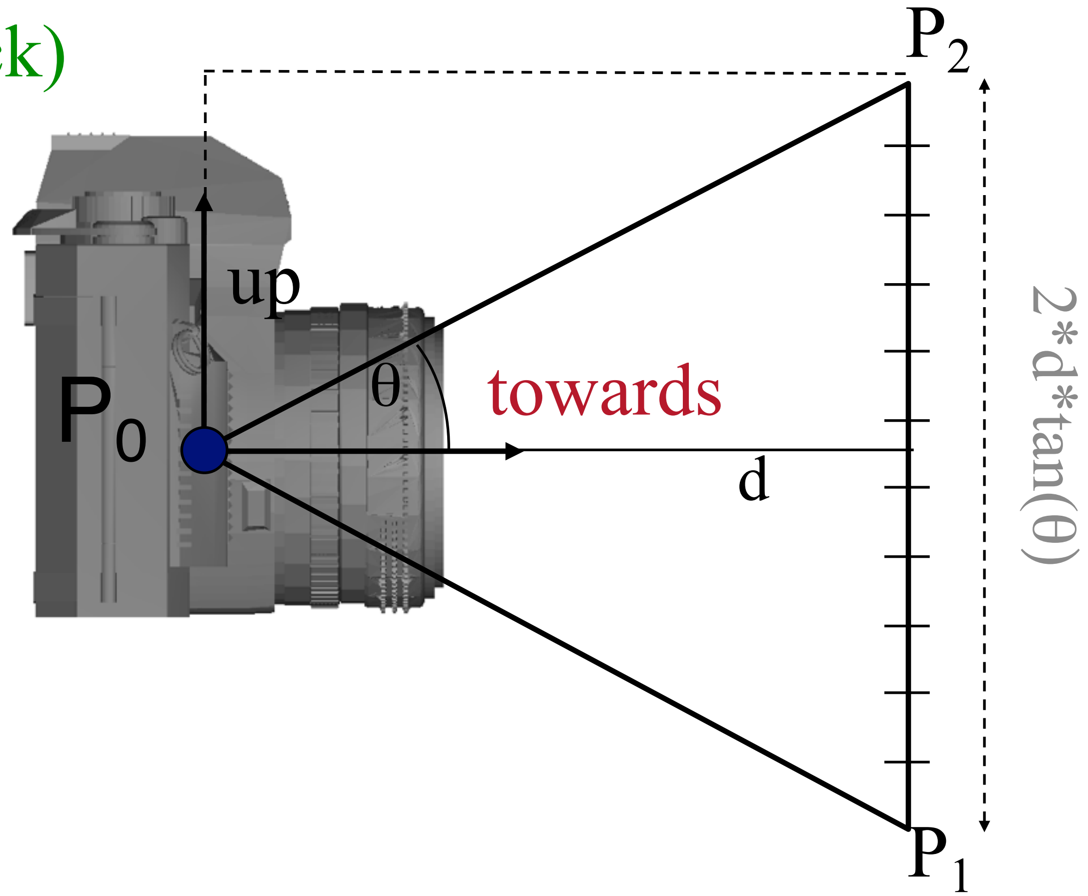
Constructing Ray Through a Pixel

- 2D Example: Side view of camera at P_0
  - What is the position of the *i*-th pixel P[i]?

θ = frustum half-angle (given), or field of view
d = distance to view plane (arbitrary = you pick)

$P_1 = P_0 + d*towards - d*\tan(\theta)*up$

$P_2 = P_0 + d*towards + d*\tan(\theta)*up$



Constructing Ray Through a Pixel

- 2D Example: Side view of camera at P_0
  - What is the position of the *i*-th pixel?

θ = frustum half-angle (given), or field of view

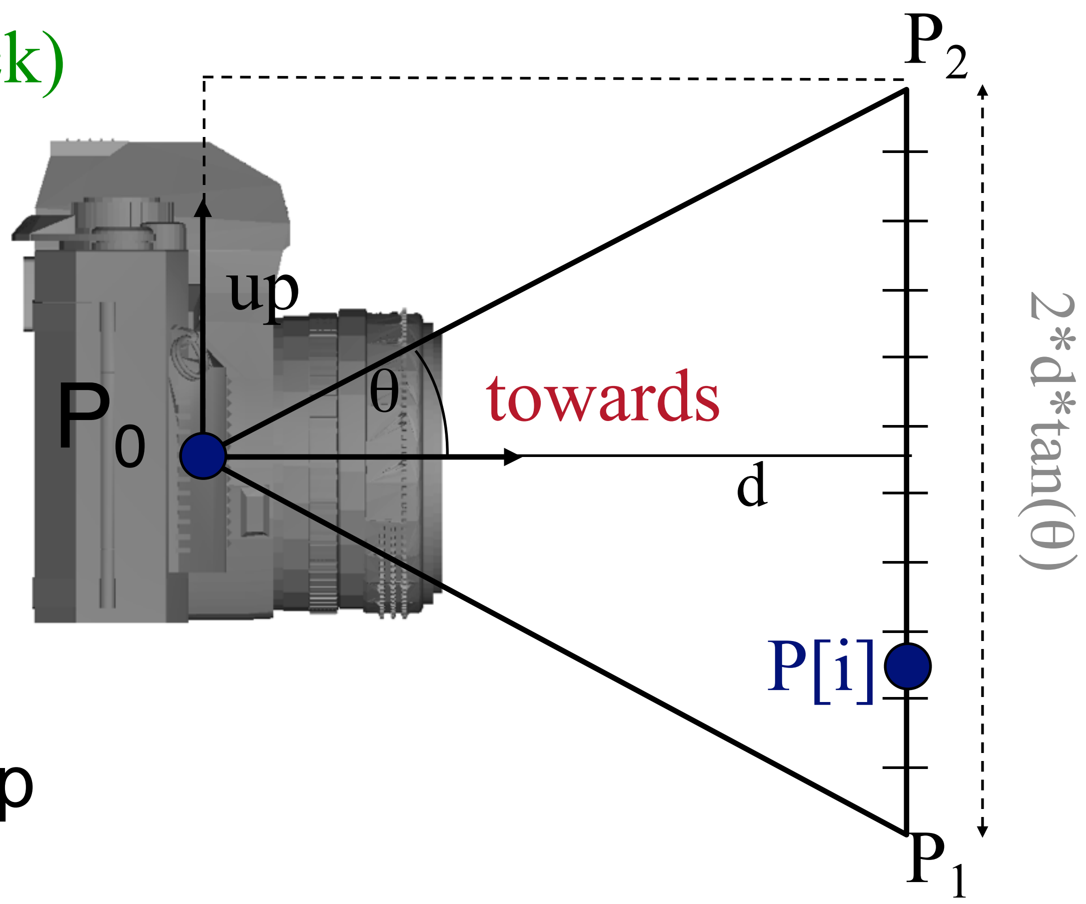
d = distance to view plane (arbitrary = you pick)

$P_1 = P_0 + d*towards - d*\tan(\theta)*up$

$P_2 = P_0 + d*towards + d*\tan(\theta)*up$

$P[i] = P_1 + ((i+0.5)/height)*(P_2-P_1)$

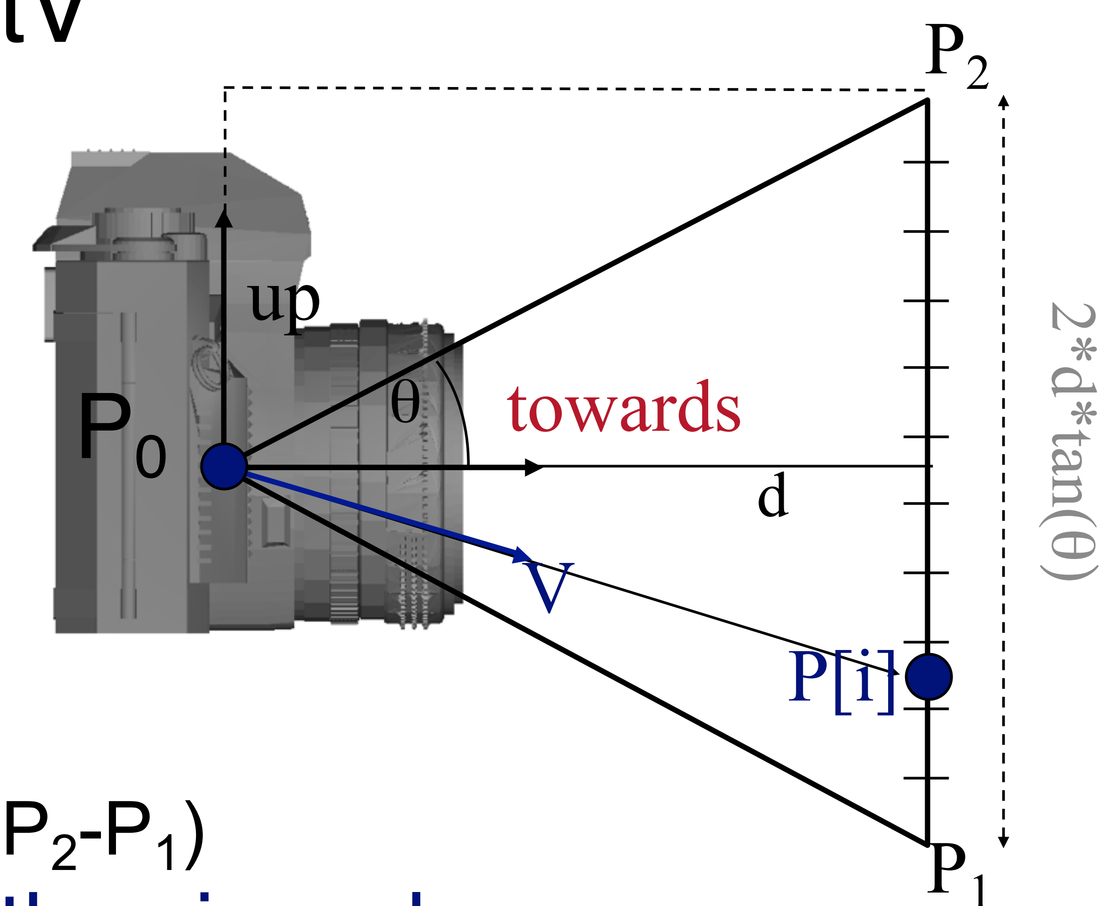
$\quad = P_1 + ((i+0.5)/height)*2*d*\tan(\theta)*up$



Constructing Ray Through a Pixel

- 2D Example:
  - The ray passing through the *i*-th pixel is defined by:

$$Ray = P_0 + tV$$



- Where:
  - P_0 is the camera position
  - V is the direction to the *i*-th pixel: $V=(P[i]-P_0)/||P[i]-P_0||$
  - P[i] is the *i*-th pixel location:

$$P[i] = P_1 + ((i+0.5)/height)*(P_2-P_1)$$

  - P_1 and P_2 are the endpoints of the view plane:

$$P_1 = P_0 + d*towards - d*\tan(\theta)*up$$

$$P_2 = P_0 + d*towards + d*\tan(\theta)*up$$

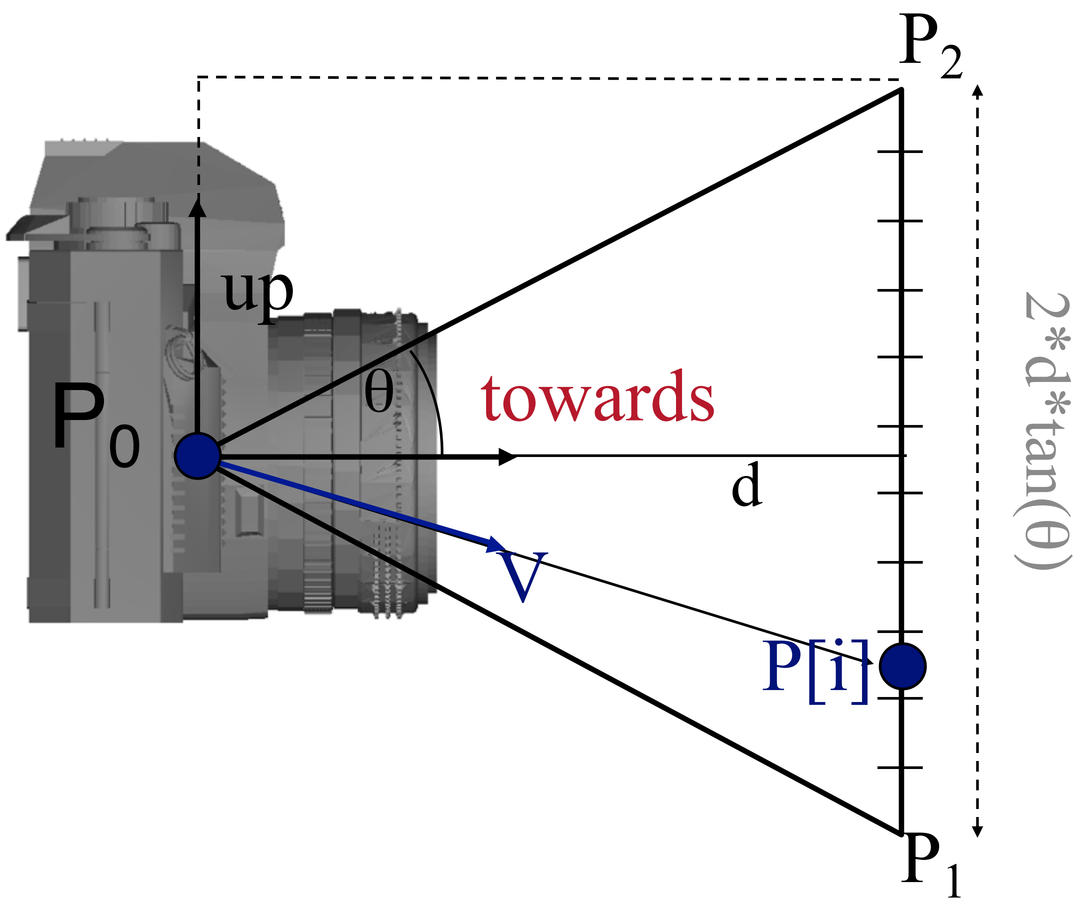
Ray Casting

- **2D implementation**:

```
Image RayCast(Camera camera, Scene scene, int width, int height)
{
    Image image = new Image(width, height);
    for (int i = 0; i < height; i++) {
        Ray ray = ConstructRayThroughPixel(camera, i, height);
        Intersection hit = FindIntersection(ray, scene);
        image[i][height] = GetColor(hit);
    }
    return image;
}
```

Constructing Ray Through a Pixel

- Figuring out how to do this in 3D is assignment 2



Ray Casting

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```

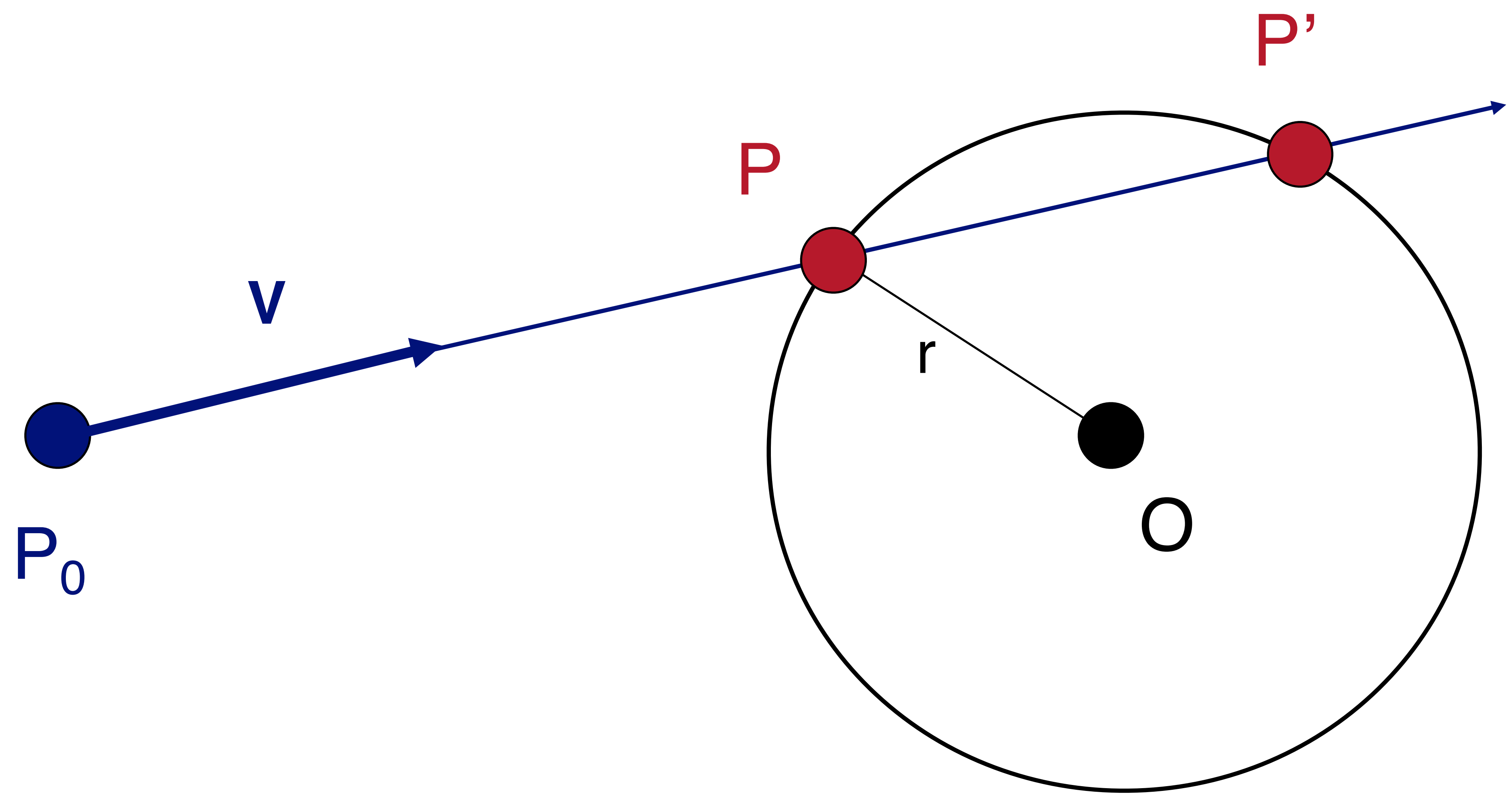
Ray-Scene Intersection

- Intersections with geometric primitives
  - Sphere
  - Triangle
- Acceleration techniques
  - Bounding volume hierarchies
  - Spatial partitions
    - Uniform (Voxel) grids
    - Octrees
    - BSP trees

Ray-Sphere Intersection

Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$



Ray-Sphere Intersection I

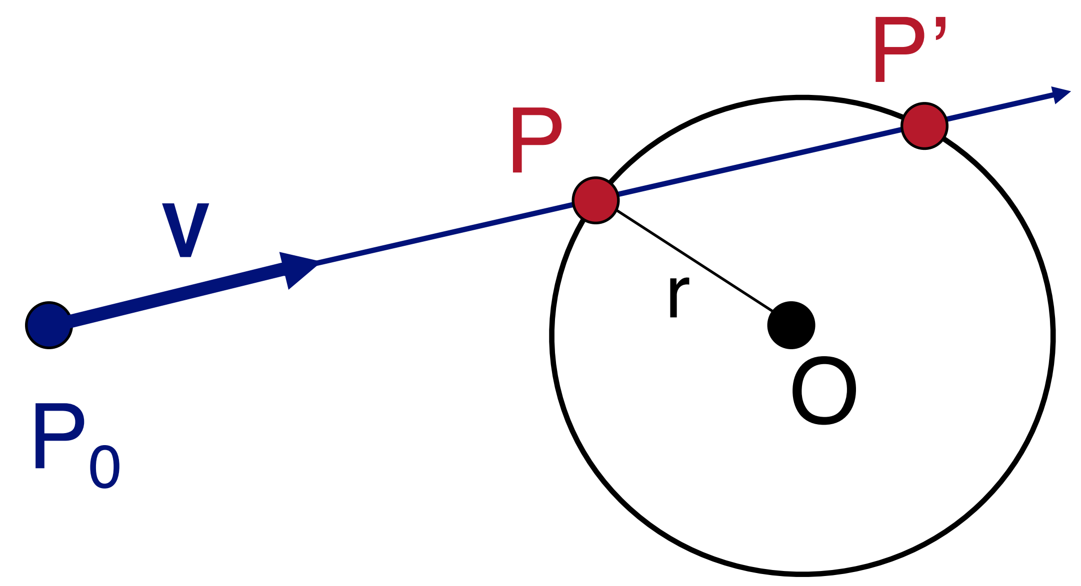
Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$

Algebraic Method

Substituting for P, we get:

$$|\mathbf{P_0 + tV} - O|^2 - r^2 = 0$$



Ray-Sphere Intersection I

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Sphere: $|P - O|^2 - r^2 = 0$

Algebraic Method

Substituting for P, we get:

$$|\mathbf{P_0 + tV} - O|^2 - r^2 = 0$$

Solve quadratic equation:

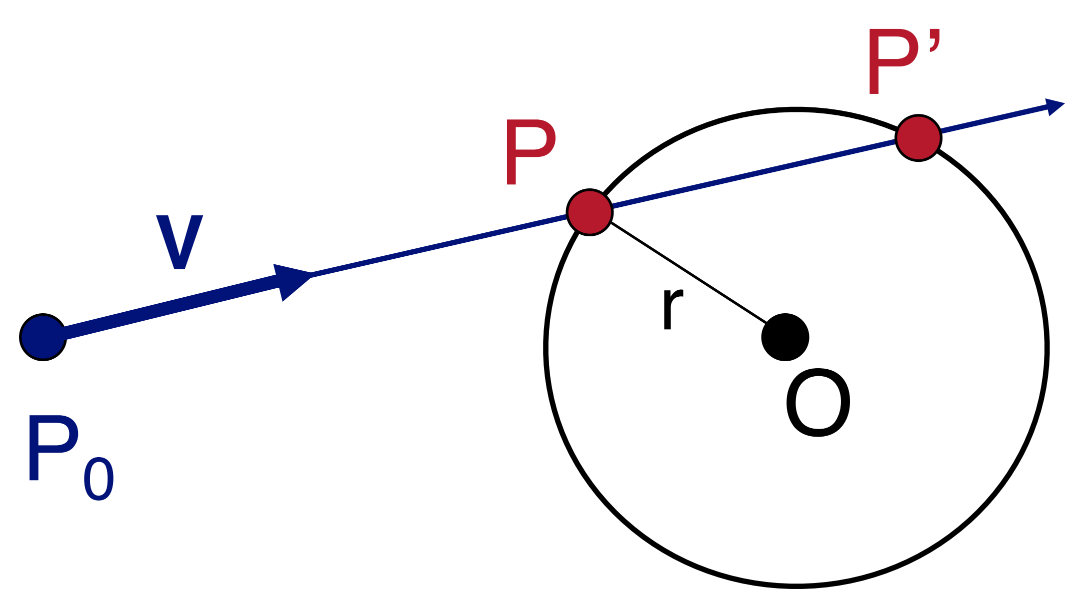
$$at^2 + bt + c = 0$$

where:

$$a = 1$$

$$b = 2\, V \cdot (P_0 - O)$$

$$c = |P_0 - O|^2 - r^2 = 0$$



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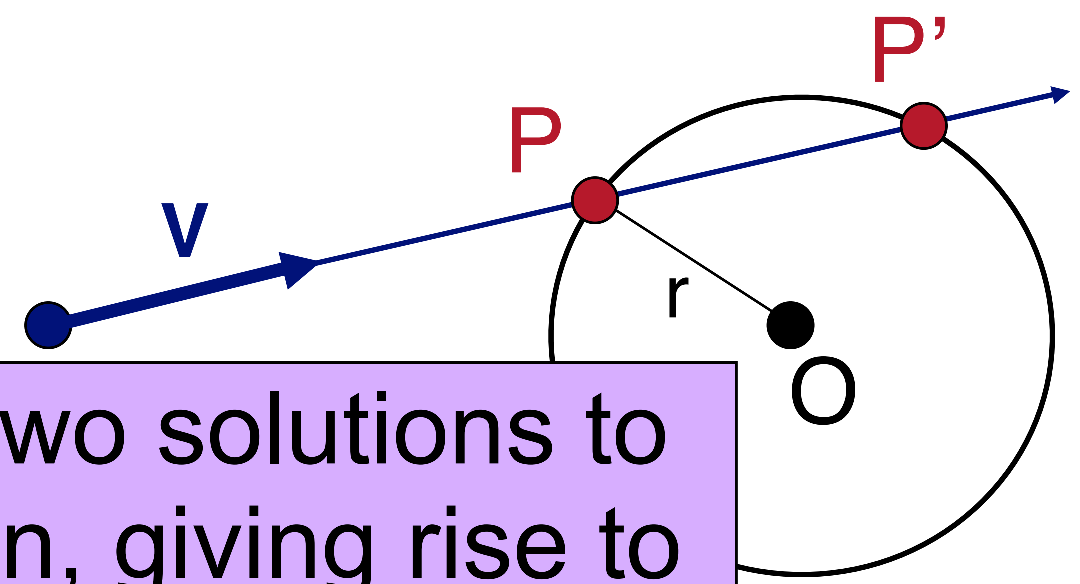
a = 1

b

c



Generally, there are two solutions to the quadratic equation, giving rise to points P and P'.
You want to return the first hit.

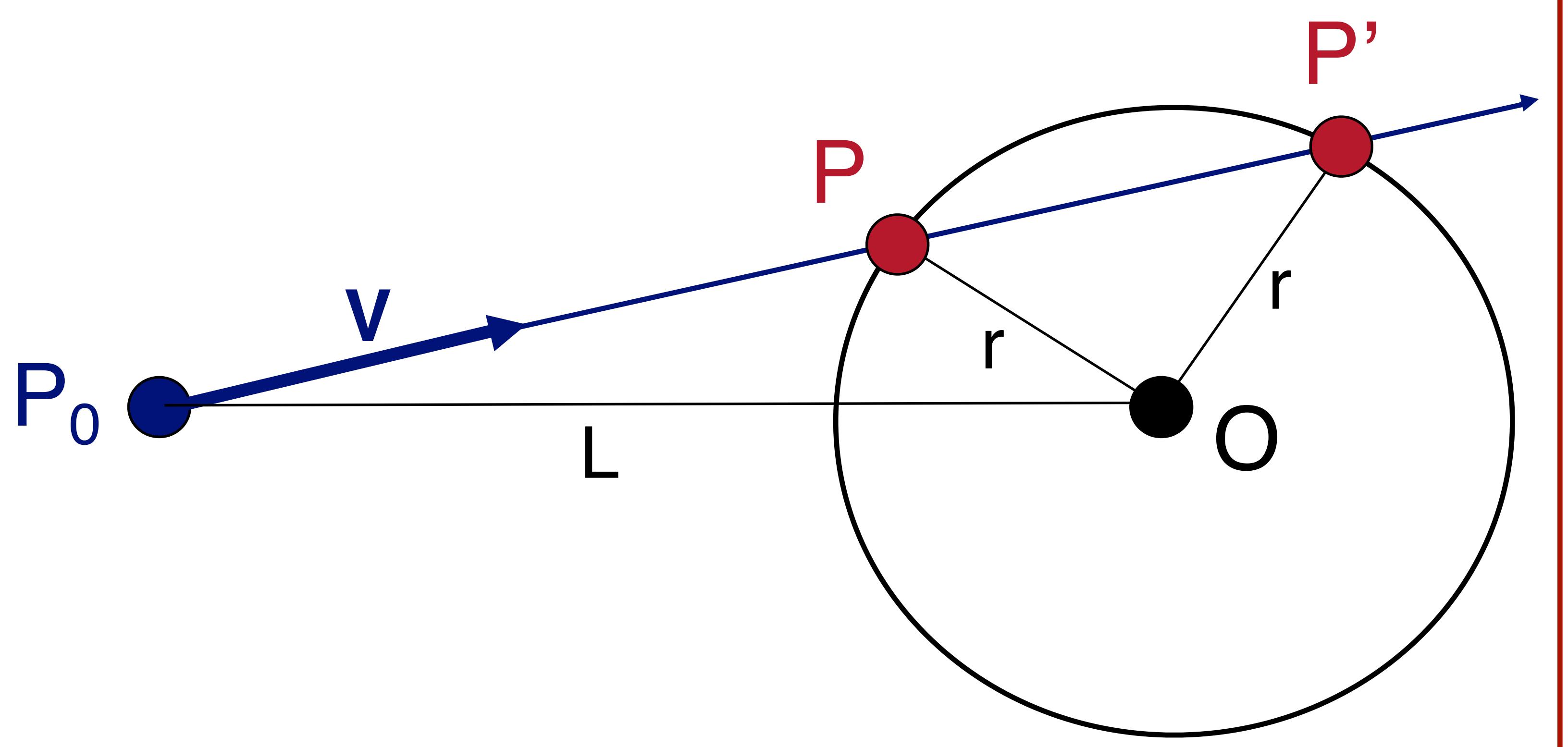
Ray-Sphere Intersection II

Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

$L = O - P_0$



Ray-Sphere Intersection II

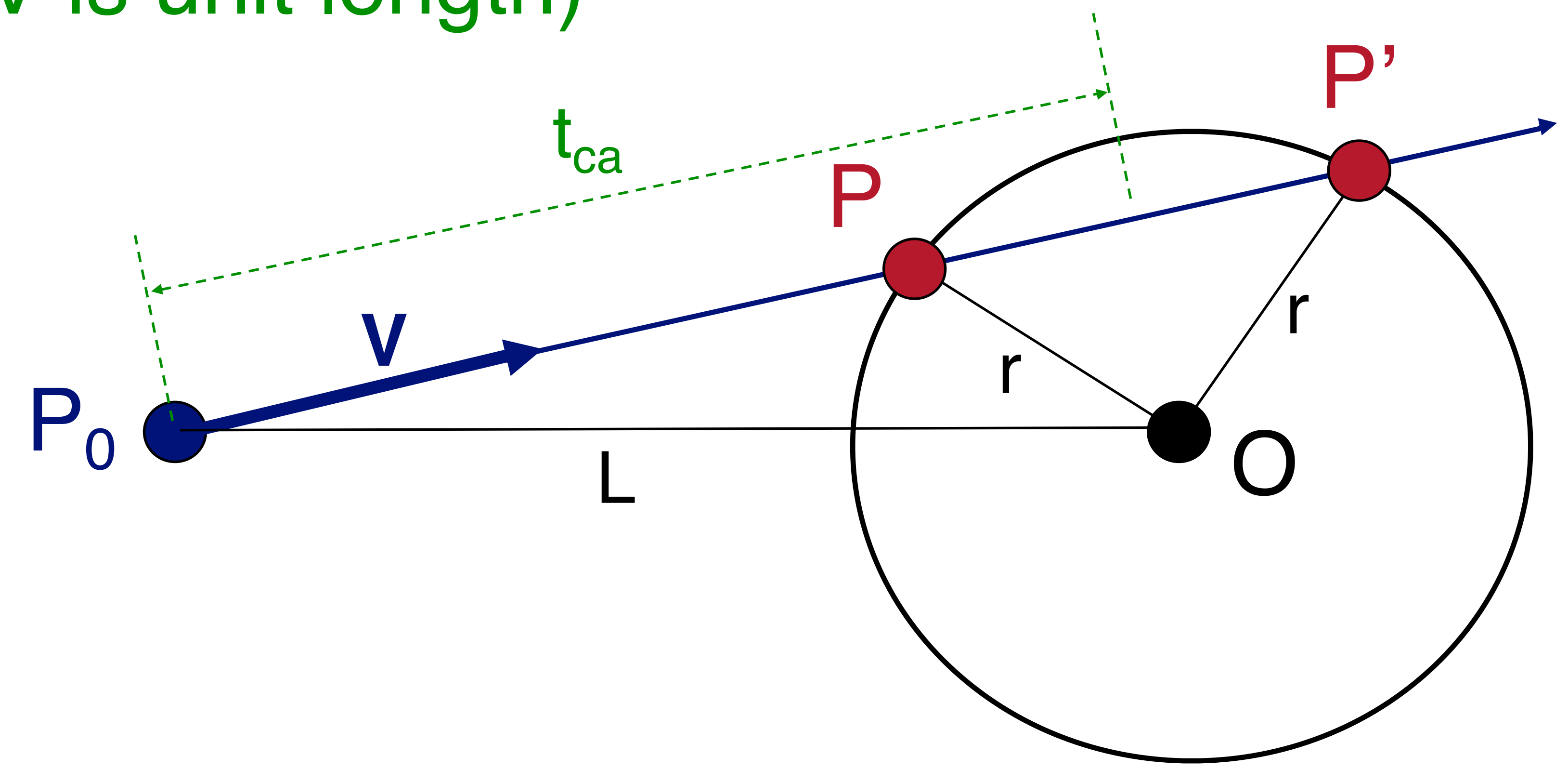
Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

$L = O - P_0$

$t_{ca} = L \cdot V$ (assumes V is unit length)



Ray-Sphere Intersection II

Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$

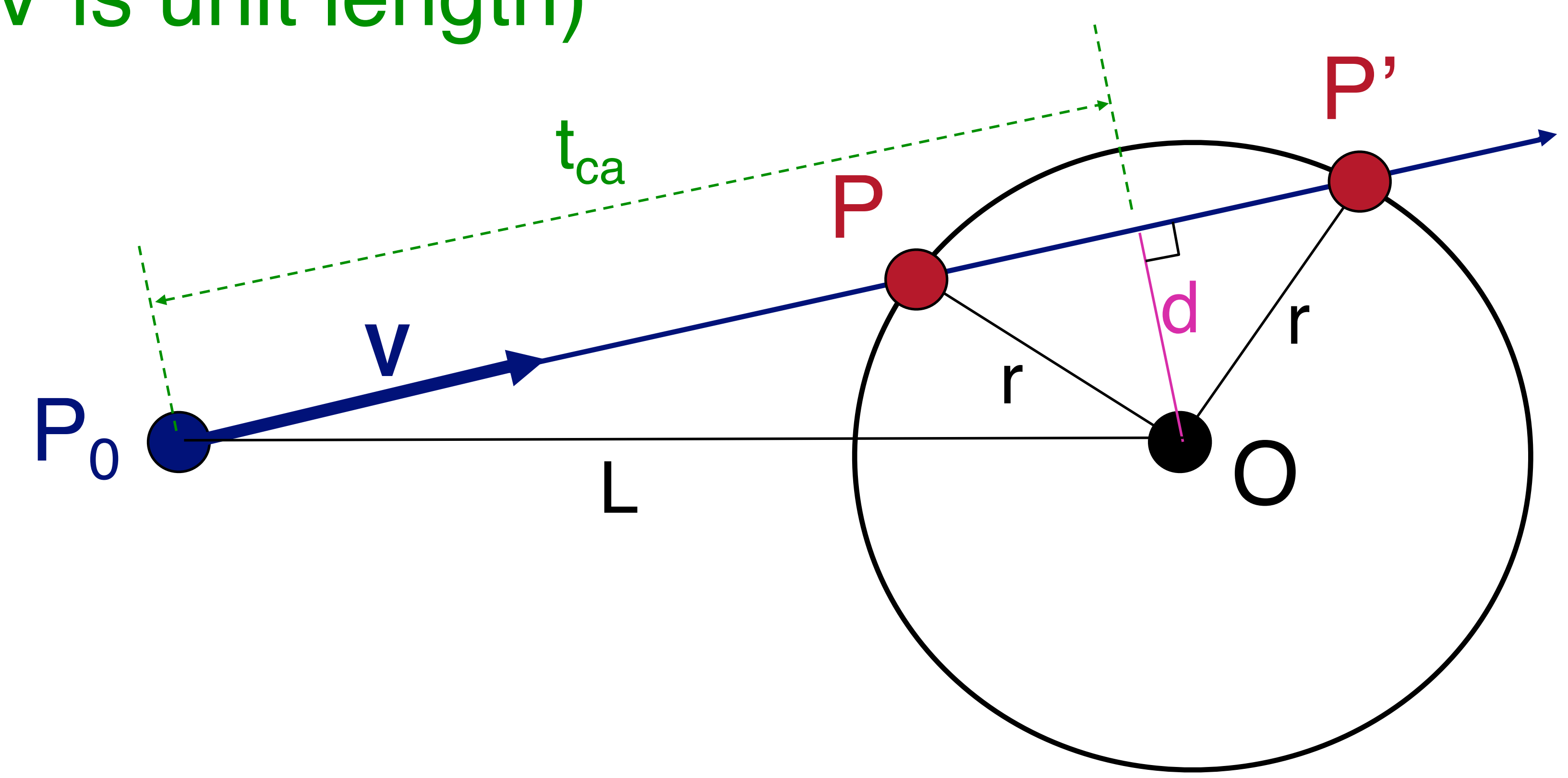
Geometric Method

$L = O - P_0$

$t_{ca} = L \cdot V$ (assumes V is unit length)

$d^2 = L \cdot L - t_{ca}^2$

if ($d^2 > r^2$) return 0



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Ray: $P = P_0 + tV$

Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

$L = O - P_0$

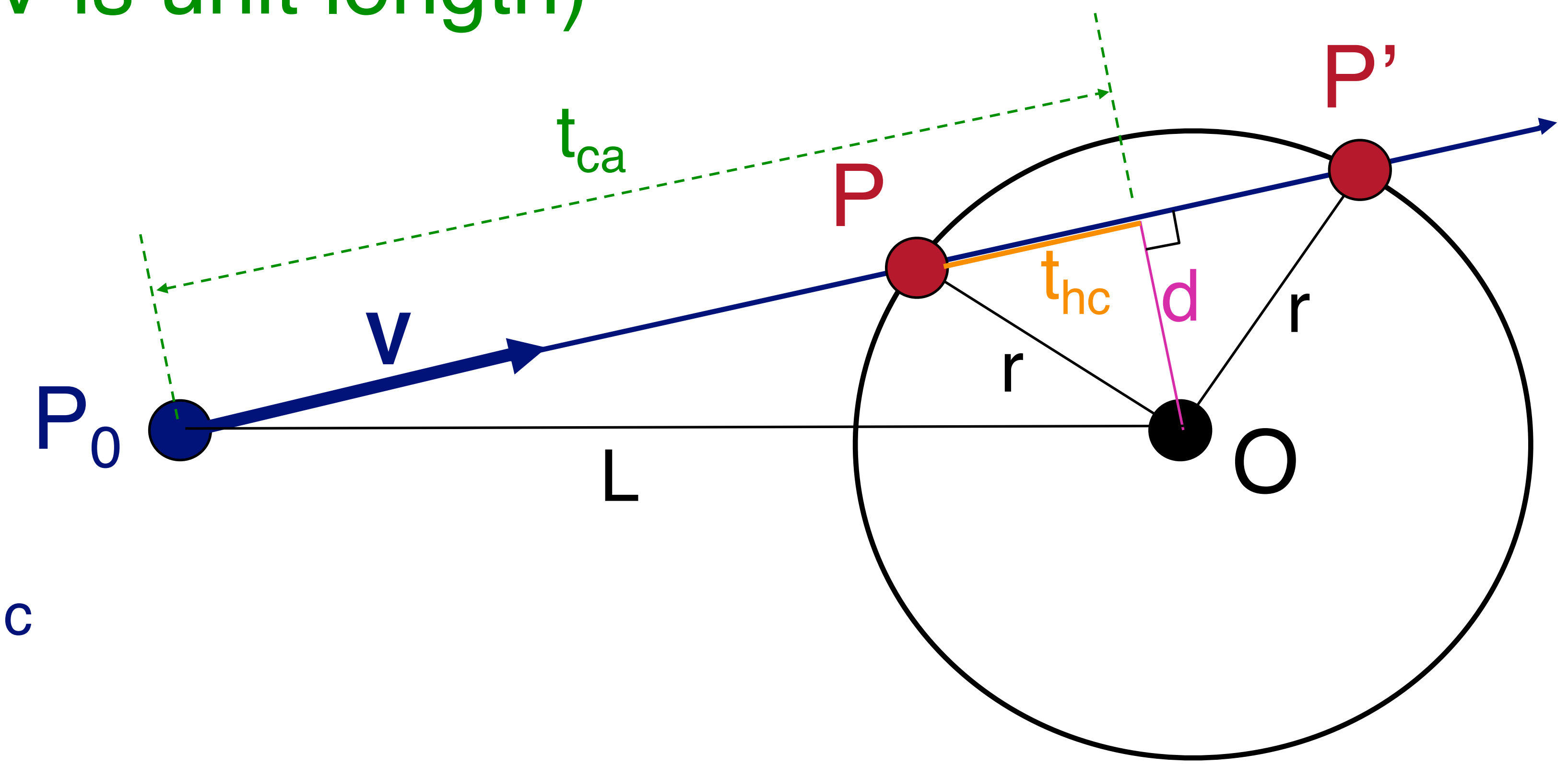
$t_{ca} = L \cdot V$ (assumes V is unit length)

$d^2 = L \cdot L - t_{ca}^2$

if $(d^2 > r^2)$ return 0

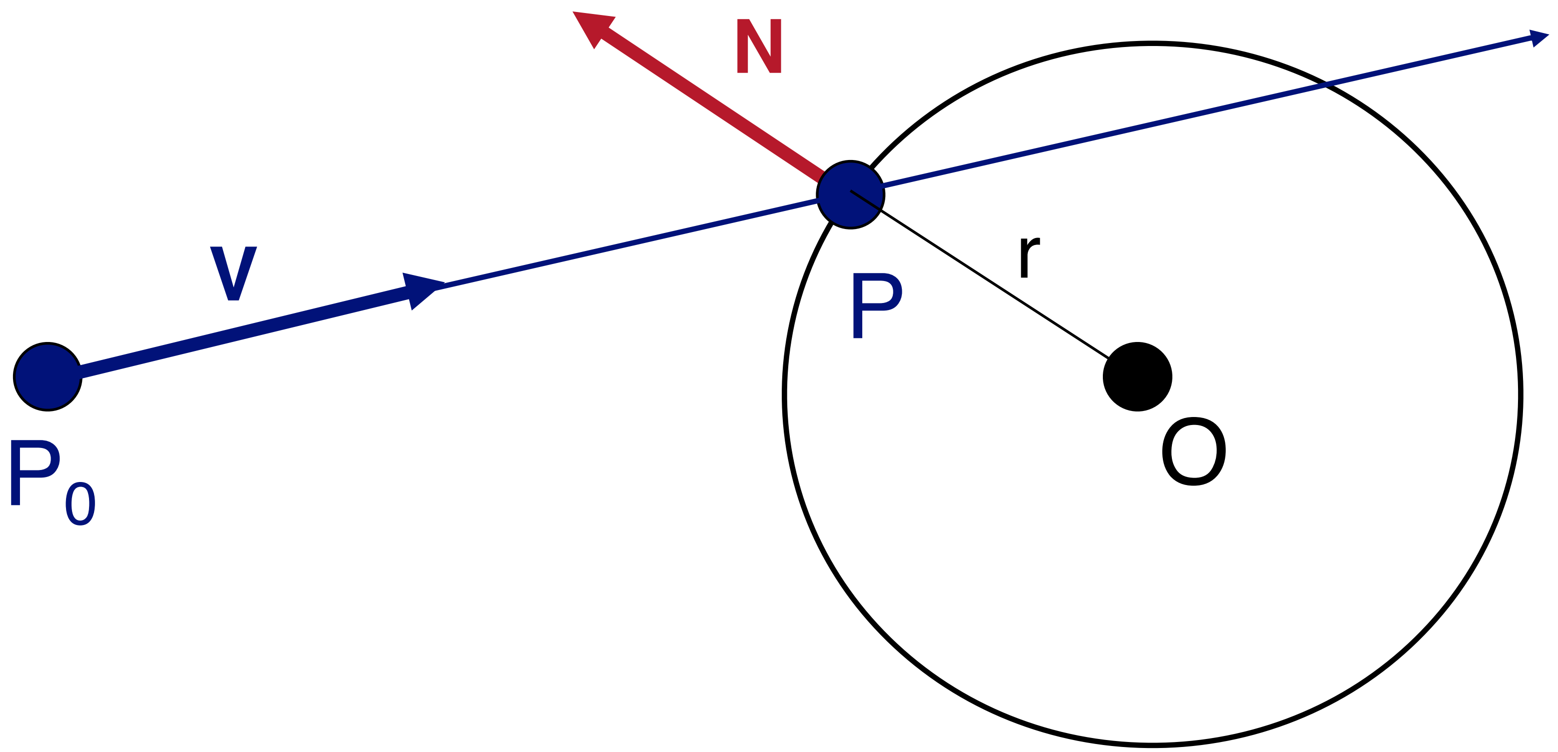
$t_{hc} = \text{sqrt}(r^2 - d^2)$

$t = t_{ca} - t_{hc}$ and $t_{ca} + t_{hc}$



Ray-Sphere Intersection

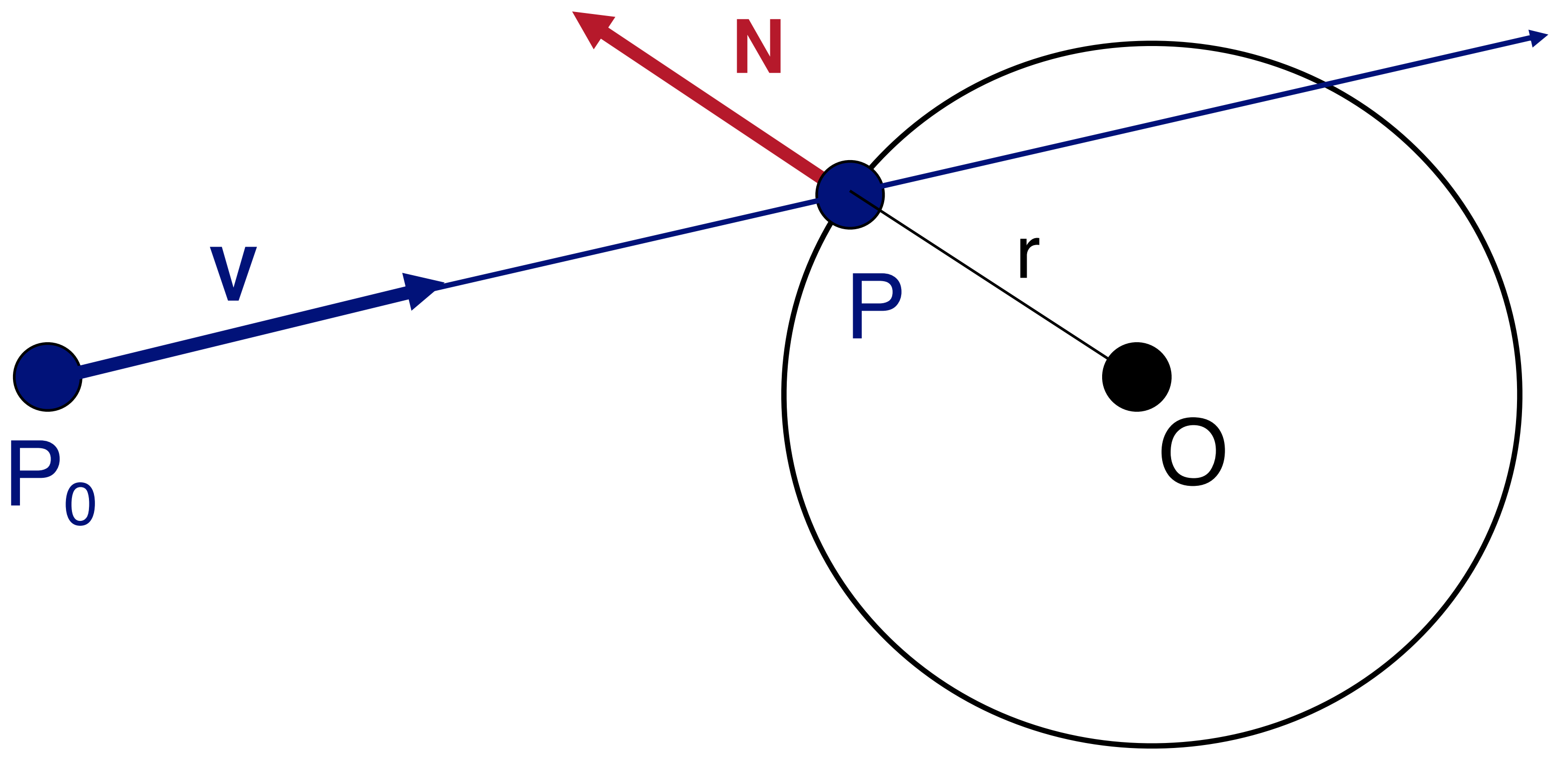
- Need normal vector at intersection for lighting calculations



Ray-Sphere Intersection

- Need normal vector at intersection for lighting calculations

$$N = (P - O) / ||P - O||$$

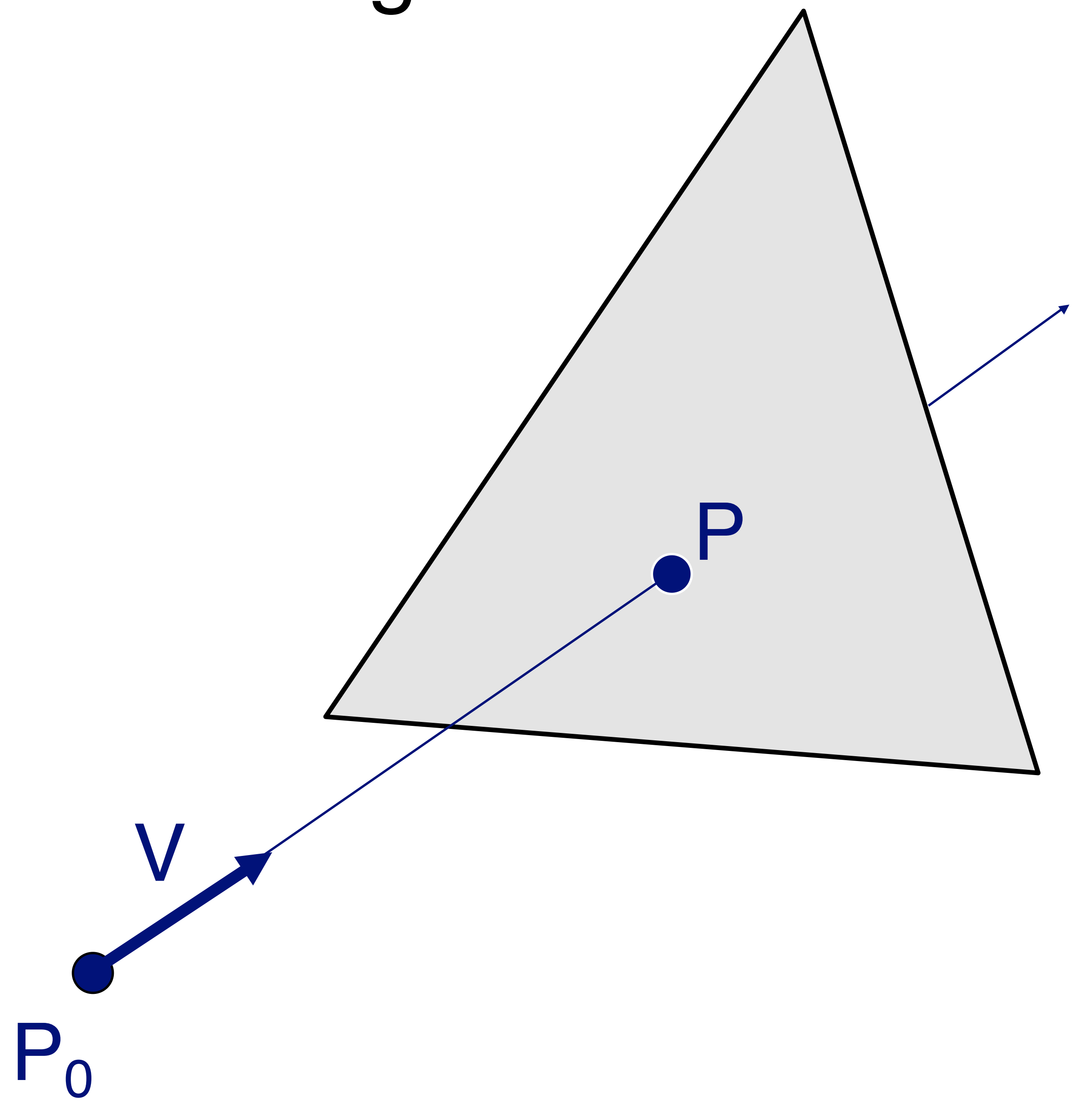


Ray-Scene Intersection

- Intersections with geometric primitives
  - Sphere
    - » Triangle
- Acceleration techniques
  - Bounding volume hierarchies
  - Spatial partitions
    - » Uniform grids
    - » Octrees
    - » BSP trees

Ray-Triangle Intersection

- First, intersect ray with plane
- Then, check if point is inside triangle



Ray-Plane Intersection

Ray: $P = P_0 + tV$

Plane: $P \cdot N + d = 0$

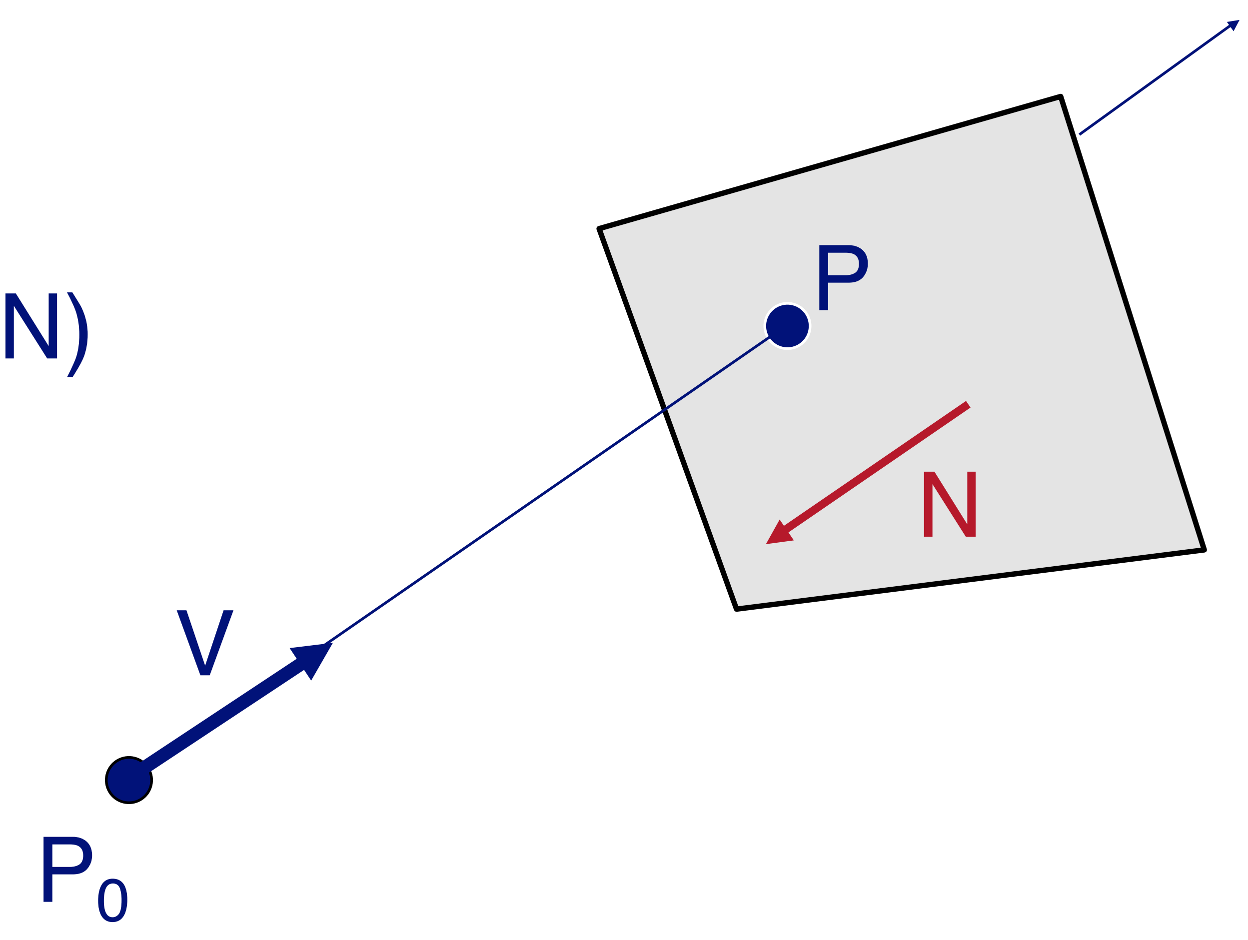
Algebraic Method

Substituting for P, we get:

$$(\mathbf{P_0 + tV}) \cdot N + d = 0$$

Solution:

$$t = -(P_0 \cdot N + d) / (V \cdot N)$$



Ray-Triangle Intersection I

- Check if point is inside triangle algebraically

For each side of triangle

$V_1 = T_1 - P_0$

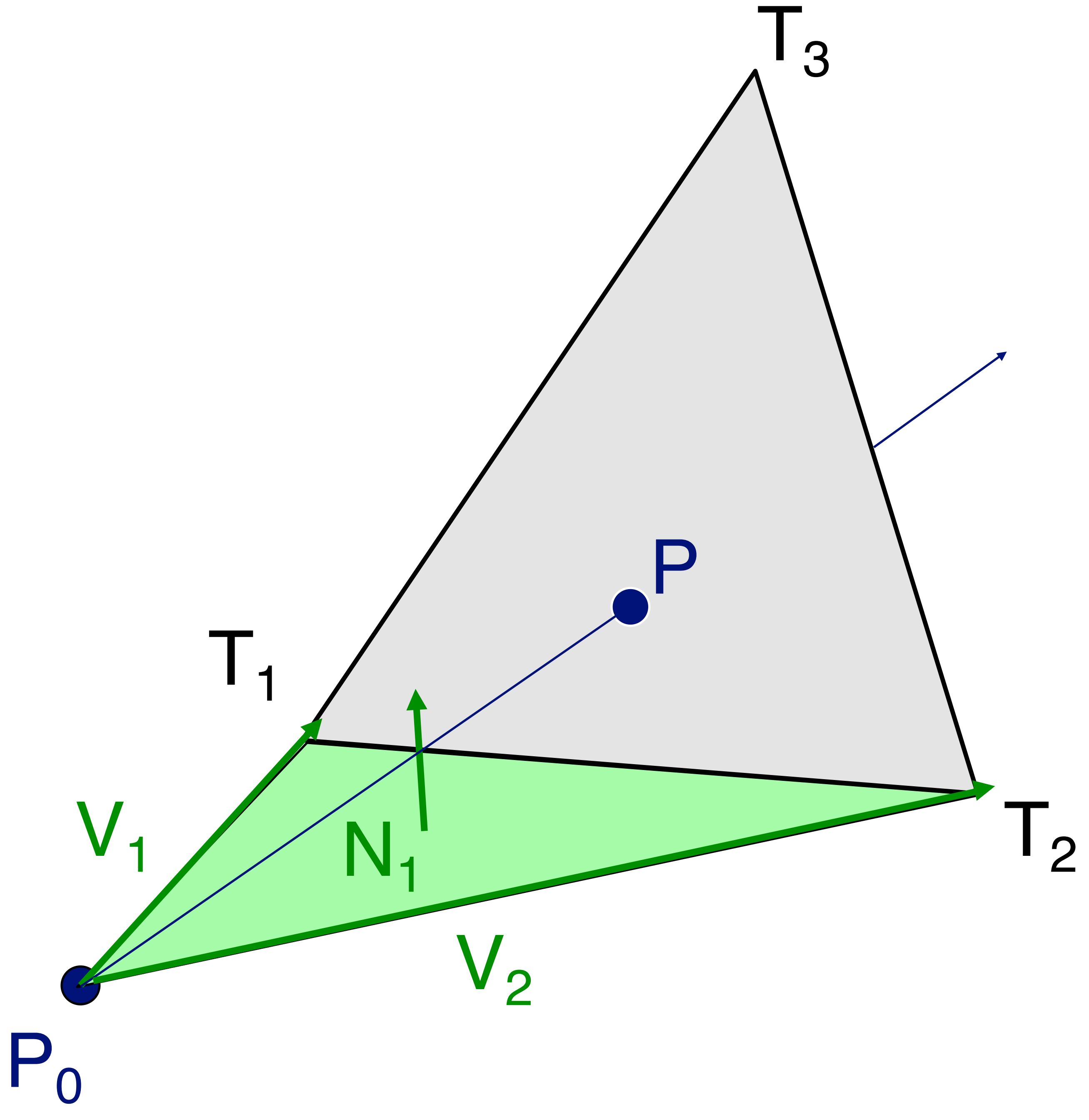
$V_2 = T_2 - P_0$

$N_1 = V_2 \times V_1$

if $((P - P_0) \cdot N_1 < 0)$

return FALSE;

end



Ray-Triangle Intersection II

- Check if point is inside triangle parametrically

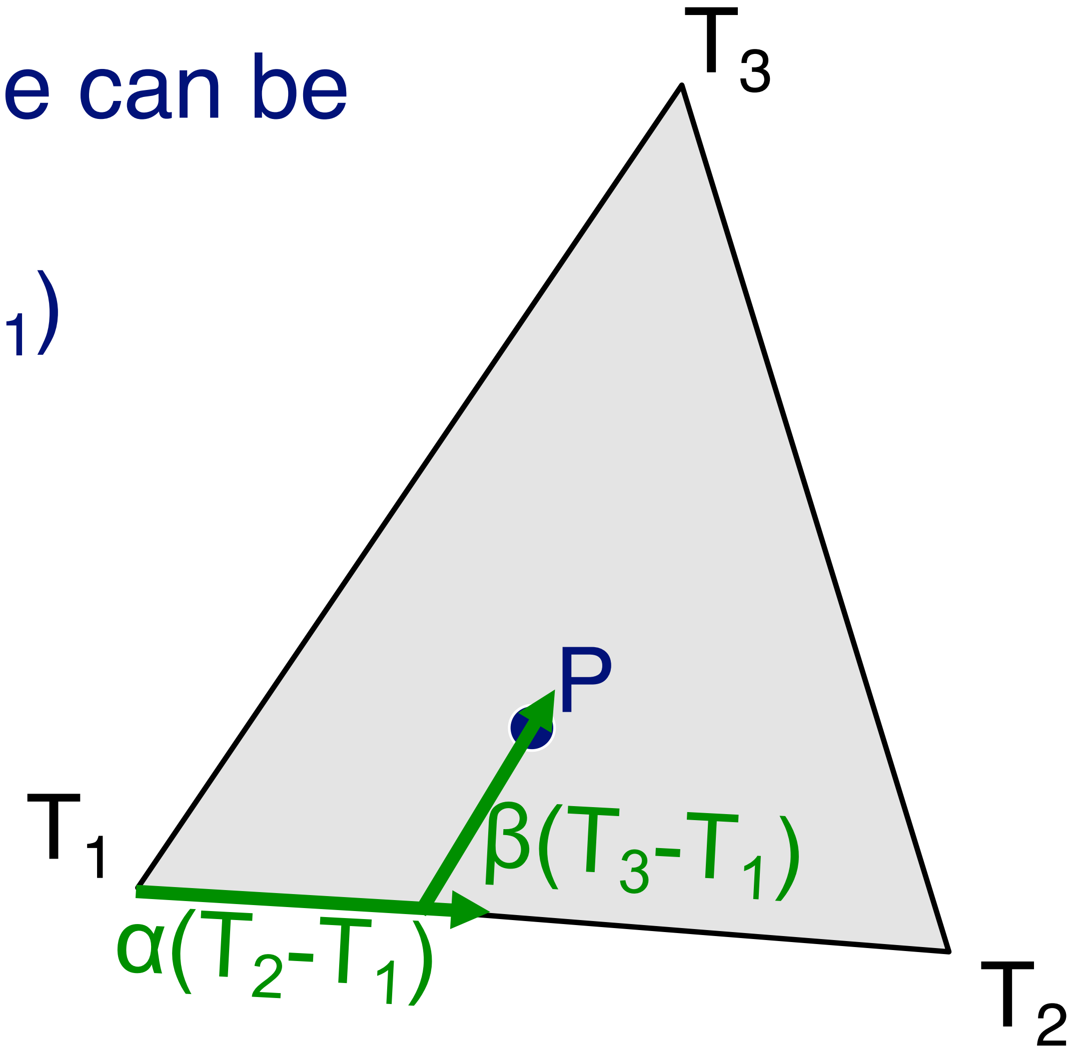
Every point P inside the triangle can be expressed as:

$$P = T_1 + \alpha\,(T_2 - T_1) + \beta\,(T_3 - T_1)$$

where:

$$0 \le \alpha \le 1 \text{ and } 0 \le \beta \le 1$$

$$\alpha + \beta \le 1$$



Ray-Triangle Intersection II

- Check if point is inside triangle parametrically

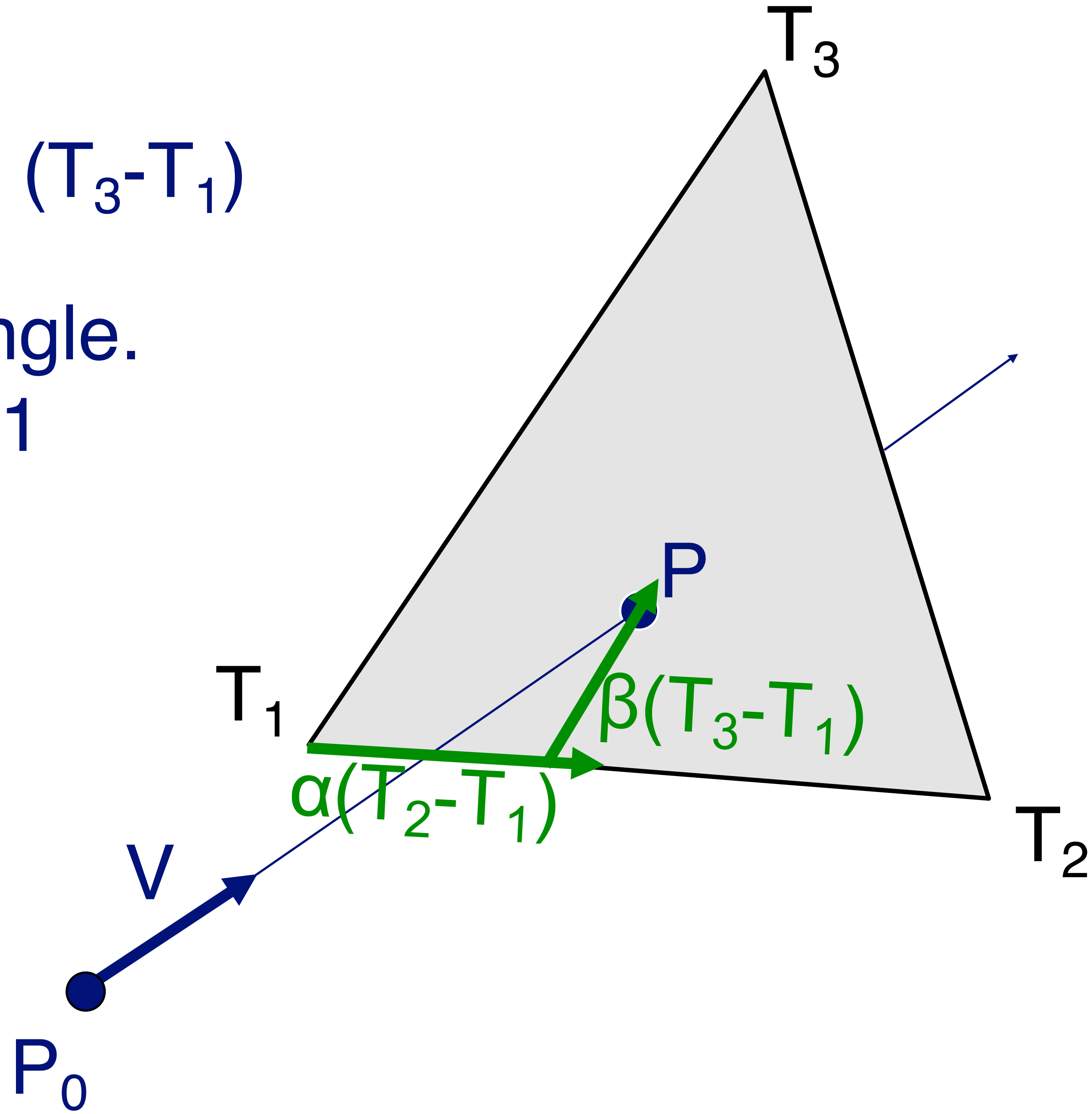
Solve for α, β such that:

$$P = T_1 + \alpha\,(T_2\text{-}T_1) + \beta\,(T_3\text{-}T_1)$$

Check if point inside triangle.

$0 \le \alpha \le 1$ and $0 \le \beta \le 1$

$\alpha + \beta \le 1$



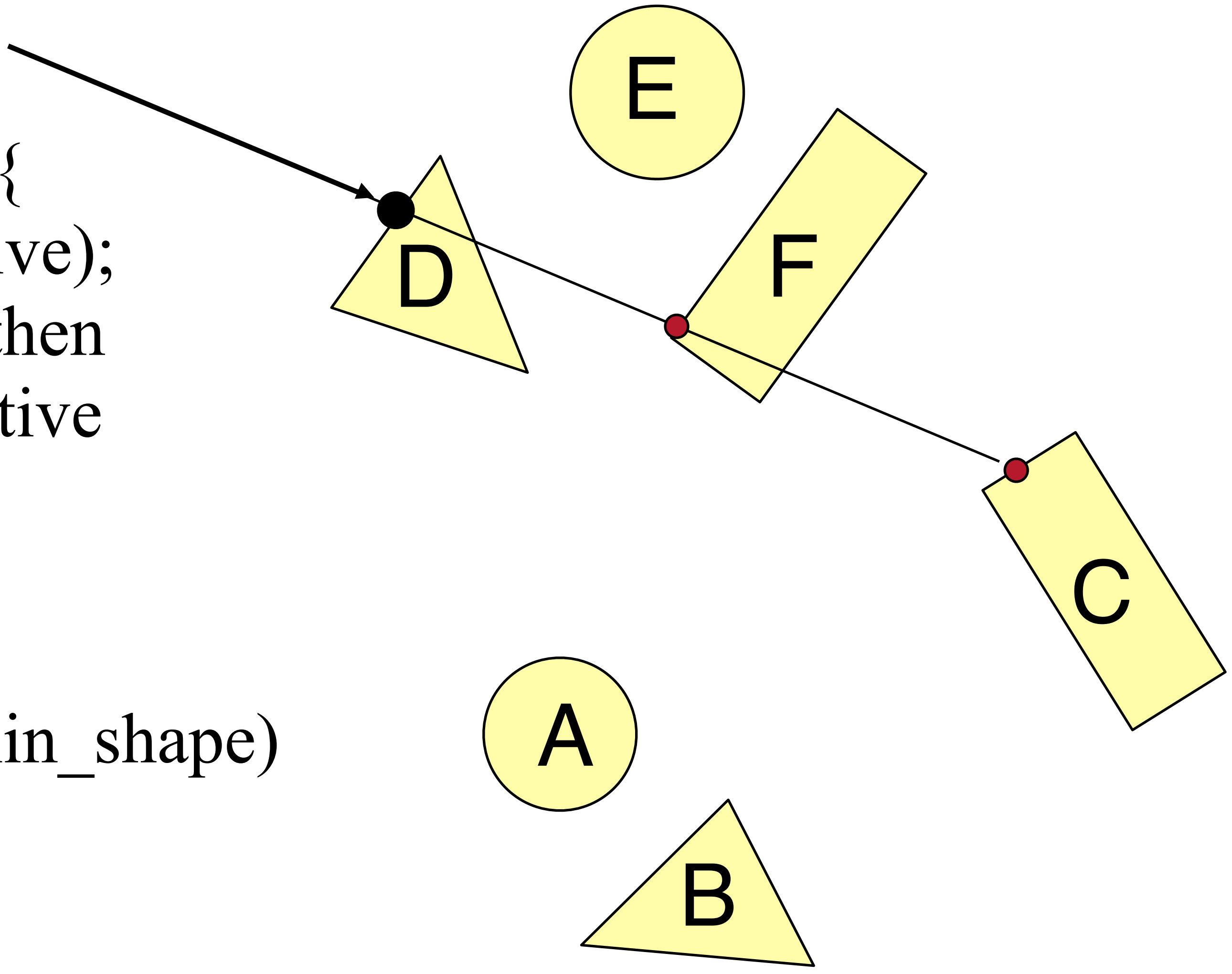
Other Ray-Primitive Intersections

- Cone, cylinder, ellipsoid:
  - Similar to sphere
- Box
  - Intersect 3 front-facing planes, return closest
- Convex polygon
  - Same as triangle (check point-in-polygon algebraically)
- Concave polygon
  - Same plane intersection
  - More complex point-in-polygon test

Ray-Scene Intersection

- Find intersection with front-most primitive in group

```
Intersection FindIntersection(Ray ray, Scene scene)
{
	min_t = ∞
	min_shape = NULL
	For each primitive in scene {
		t = Intersect(ray, primitive);
		if (t > 0 and t < min_t) then
			min_shape = primitive
			min_t = t
		}
	}
	return Intersection(min_t, min_shape)
}
```



Next Lecture

- Intersections with geometric primitives
  - Sphere
  - Triangle

- Acceleration techniques
  - Bounding volume hierarchies
  - Spatial partitions
    - Uniform grids
    - Octrees
    - BSP trees